Pythagorean Mathematics, the Logos and the Liberal Arts

" Enthusiasmus ist die Mathematik. Ohne Enthusiasmus, keine Mathematik"
'Enthusiasm[1] is mathematics. No enthusiasm, no mathematics' Novalis

This paper is a continuation from my paper in the ANPA 2020 Proceedings , 'Towards a 21st century Pythagorean Mathematics'[2] where I began to look at the development of European mathematics from its Pythagorean origins in the 6th century BC in Greece until now. Clearly there are interesting and important stories of other mathematics from other parts of the world[3]. But it is European mathematics which has come to be our global mathematics. When we delve into its history, we discover an inherent holism in Pythagorean mathematics which no longer exists today. We can begin to uncover where and how this was lost, why this is important now and how we might redress the balance.

I shall not attempt to summarise the 2020 paper. I shall refer to it where needed.

Our present notion of mathematics has only half the scope of the original Pythagorean mathematics. Using Iain McGilchrist's analysis of the different stances and aptitudes of the two different hemispheres of the cerebral cortex[4], it appears that our present, standard mathematics corresponds mostly to left brain activity.

We could characterise these aspects in mathematics, approximately thus:

Left brain	Right brain
Quantitative	Qualitative
Visual	Musical
Abstract, technical	Spiritual, meaningful.

Pythagorean mathematics encompasses both[5].

Martin Buber's[6] perspective is also helpful here. He identified two main, human modes of being in the world. The first is where we relate directly with Other,

[1] Etymological root of 'enthusiasm', 'in God'.
[2] NGG 'Towards a 21st Century Pythagorean Mathematics', 'Aporia' ANPA Proc. 2020
[3] For example, Chinese mathematics or Mayan mathematics.
[4] Iain McGilchrist, 'The Master and His Emissary, The Divided Brain and the Making of the Western World', Yale Univ.Pr. 2009.
[5] Clearly, these correspondences are with tendencies of the left and right hemispheres. Both hemispheres are involved in different ways in all mathematics.
[6] Martin Buber, 'I and Thou', transl. R.G.Smith, publ. Continuum, 1937,1958. 1st German ed'n, 'Ich und Du', publ. 1923

with not-I. As individuals we are each in a great not-I. We might call this 'the world' or 'the universe' or the 'Divine' or…. In this direct relationship, we may address the great Other, of which we are part, as 'Thou'. Within this Other, we perceive other others, trees, birds, humans, myriad others. We might address them as 'thou'[7]. These are 2nd person relationships.

Martin Buber points out that in these relationships with Other and others, there is a living dialectic: both parties and the relationship between, can all change, evolve. To the extent that we recognise the livingness of other, we are called into being more alive. He contrasts this with our 3rd person relationships with the world, with other beings, where we think of other as 'it'.

When we think about things, we are separate from them. We withdraw into our minds[8]. This is a different mode of being from relating to others, being with others. In this mode we are not only thinking but also sensing, feeling the other, feeling ourselves in relation to the other, and to varying extents, aware of the relationship[9].

Buber named these modes, 'I-*Thou'* or 'I-*thou'* and 'I-*it*', as our 'I' has a different nature in the two different modes. The stance of the left brain creates the 3rd person world of *It,* the world of consciousness, communication and control. We need this for the smooth running of our material, social reality. Our right brain corresponds more to the '*thou'* world where we are communing with other; communion opens possibility[10]. Our 'I-*thou'* is in living flux.

Our consensus language is predominantly in the '*it*' world. Paradoxically although we need this to communicate with each other, it can also separate us from each other, and from reality[11]. This paradox is at the heart of being

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[7] Following the example of St.Francis.

[8] We may have emotions about the object of our thinking but this is our individual affair; they are not relational; we are not reaching out with our feeling to sense the other's being, It is strange for us to try this with other-than-human beings but it is possible. There are simple techniques to help us begin, for example, saying "Hello" or "Thank you". When we say "Hello", we need to give the other being time for us to attempt to open our sensory awareness to perceive any response. "Thank you" can be one way to acknowledge that we are then taking our attention elsewhere, into a different mode.

[9] This is often the case in our meetings with others; they are formulaic, not real meetings. We have underlying, background mind talk about the other and the situation. When we attempt to really relate to people with learning differences aka learning disabilities, we cannot rely on our normal, social customs which allow us to glide through encounters without really meeting. We are called to be in our deeper selves.

[10] Heidegger considered possibility to be the essence of being human. Possibility implies free will and creativity. 'Being and Time', transl. Joan Stambaugh, publ. State Univ. NY Press 2010. Original publ. 1927

[11] Rainer Maria Rilke, "*With all its eyes the natural world looks out into the Open. Only our eyes are turned backward, and surround plant, animal, child, like traps as they emerge into their freedom. We know what's really out there only from the animals' gaze; for we take the very young child and force it around, so that it sees*

human. It involves the heart. The left brain stance is head thinking based. Heart thinking connects more with the right brain stance[12]. These two modes of being are the parameters of our incarnate lives. We do not solve this paradox intellectually. We live it. We navigate and dance[13] our way between the different modes, to find and create our true being within this wondrous Life.

This may seem to you a strange digression in a paper about mathematics. This is because we have inherited predominantly the left brain, abstract, technical parts of the original, Pythagorean mathematics. These concerned all aspects of human life. The aspects with more human and spiritual meaning were lost when the Pythagorean school was broken, which happened for political reasons[14].

We shall return to this wider conversation after a whizz-bang tour through a few key periods in Western mathematical history.

6th Century BC Greece

The *raison d'être* of the Pythagorean school was *philia*, loving friendship. This is the root of the original meaning of philosophy (*philosophia*), namely, the wisdom of love[15]. This was the philosophy which gave birth to their mathematics (from *ta mathemata*, those things which have been learned). Understanding this, is key to recognising the holism of Pythagorean mathematics, and why its renaissance would be beneficial now.

This was the time of the pre-Socratic philosophers, a time of a new mode of human consciousness. Until that time human culture, had been based in a world of gods and nature spirits, relationships with beings of different natures, in Buberian terminology, a world of I-*thou*. Now a new kind of thinking was beginning, a separation from nature. It was the dawn of the I-*it* world.

Within this, Pythagoras cognised numbers as a new kind of being. The Divine Oneness, the Monas, was their source. These numbers (*arithmoi*) were the

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objects – not the Open, which is so deep in animals' faces." Duino Elegies, no.VII, transl. Stephen Mitchell, publ. Random House 1982

[12] See S.H.Buhner, 'The Secret Teachings of Plants, the Intelligence of the Heart in the Direct Perception of Nature' publ. `bear & Co., 2004, for the wealth of neuroscientific validation of heart thinking; See G.Kühlewind, 'Thinking of the Heart and other essays', transl. and ed. F.Scwarzkopf, Rudolf Steiner College Press,1987 for an explication of the phenomenon.

[13] Rudolf Steiner layers the 2-ness of Buber (which is also 3-ness) with 3 modes of human becoming in the world, thinking, feeling and willing/doing, corresponding to head, heart and gut. We dance this triad.

[14] Iamblichus, pp.116-7, 'The Pythagorean Sourcebook and Library', compiled and transl. K.S.Guthrie, publ. Phanes Pr. 1987

[15] NGG ANPA Proc.2020, p. 387. Note that it is the wisdom of '*philia*', loving friendship, not '*agapé*', love of the divine, nor '*eros*', romantic, sexual love, nor '*storgé*', familial love.

natural, whole numbers which could be represented by arrangements of pebbles. The tetraktys, a triangle of pebbles, one over two, over three, over four, showed their unfolding from the one. Numbers' quantitative aspects were studied as *logistiké*, the mathematics of the new *it* world of separation from, and control over, nature. A*rithmetiké*, the complement, was the study of the qualitative aspects of numbers, their forms (*eidé),* their different being natures and interrelationships. This was closer to the t*hou* world.

This mathematical knowledge was held secret within the inner, esoteric group[16], women and men, who shared all their material goods, having committed to live according to the principle of 'Love thy neighbour as thyself'[17], the wisdom of love. Their secrecy protected the powerful knowledge from being misused by people with less worthy motives. When the Pythagorean community was broken, knowledge of the technical mathematics was released from its alchemical vessel. The age of the sorcerer's apprentice had begun.

The Middle Ages to the Renaissance in Europe

Pythagorean mathematics and the Pythagorean ethos was revived in the 13th and 14th centuries AD within the teachings of the 7 Liberal Arts in the school of Chartres Cathedral[18]. These comprised both verbal and mathematical arts; the Trivium, grammar, logic and rhetoric, and the Quadrivium, number, geometry, music, astronomy.

Ironically, at this time, the quasi-nemesis of Pythagorean teaching was entering Europe via Middle Eastern trade (a by-product of the Crusades) in the form of the symbol for zero[19]. This ushered in a revolutionary, new numeral system. The cumbersome, Roman numerals were gradually replaced by the Hindu-Arabic, decimal place system, based on the abacus, designed for ease of calculation[20]. This led to decimal fractions[21] which then led to the wholly new concept of the number line[22].

During this period and beyond, mathematical texts were becoming generally more symbolic, less embedded in the lingua franca (at that time Latin). Letters

[16] Ibid. p.391
[17] Iamblichus, pp.112-4, ibid. We could see them as a quasi-Christian, pre-Christian community, or Buddhist but it was Platonism (adulterated Pythagoreanism?) which informed subsequent Christianity.
[18] NGG, ANPA Proc.2020, p.405
[19] NGG, ibid. pp.405-6
[20] NGG, ibid. p.406
[21] Before that, fractions were a form of ratios of whole numbers.
[22]NGG, ibid. pp.46-7 and NGG, 'Historical Changes in the Concept of Number, Mathematics and Number Theory', ANPA Proc. 2014, 'Groupings', pp.25-52

of the alphabet were being used for unknowns. And the determinate nature of 'unknowns' in equations, opened into the revolutionary 'variable' concept[23]. Change was entering formal mathematics in symbolic algebra, By approximately 1600 AD mathematics had its own language with its own grammar and syntax.[24]

The 'Enlightenment' to the 19th century in Europe

By the end of the 17th century, the concept of variable, combined with the idea of numbers as points on a line, had given birth to the differential calculus. Leibniz, Newton, Gregory, maybe others, had found ways to describe mechanical, quantitative change mathematically[25].

This wholly new mathematics of change corresponded to the new cultural idea of progress[26] and engendered mathematical, technological and economic possibilities that powered mathematics and human culture for over two centuries. By the end of the 19th and the beginning of the 20th century, the limitations of this mechanical idea of change, were becoming apparent.

20th to 21st century AD in the Western world[27]

The *fin de siècle*[28] brought new forms of art, literature and science and in mathematics, non-Euclidean geometries. In the early 20th century Hilbert's vision of constructing a formal mathematics of proven certainty was shattered by Gödel's incompleteness theorem. In science quantum physics revealed a world where there was no Newtonian, objective observer; observation was an intervention; nature was announcing boundary limits of I-*it.*[29] But the mainstream approach was to ignore the philosophical implications and instead to find ways to continue quantifying and controlling.

In mathematics new, more qualitative ideas began to emerge, including attempts to describe evolving beings and non-linear processes – topology, knot theory, cybernetics, complexity theory, catastrophe theory, chaos theory etc. So many theories, so many technological advances, so many catastrophes and so much chaos!

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[23] These were not the words used then.

[24] Martin Lowe "The Introduction and Development of Symbolic Representation in Mathematics During the Late Renaissance", ANPA Proceedings 2018,'Chiasmus', Ed. John Ceres Amson, p.315

[25] NGG, ANPA Proc. 2020, pp.408-9

[26] It was only at the end of the 16th century that the word 'progress' came to be used for "growth, development, advancement towards higher stages." It "was originally used in English solely for 'a state journey by royalty'." www.etymonline.com

[27] Which by this time includes large parts of the whole globe which have been westernised – mostly by force.

[28] The very fact that the end of the 19th century has been singled out in this way, speaks of the importance of the profound cultural changes occurring at this time.

[29] David Bohm, "Nature responds according to the way in which she is approached".

NOW

Clearly mathematics has played a key role in the societal changes of the last two and a half millennia. And the nature of mathematics itself has changed profoundly. I want to investigate this story a little closer, bearing in mind the questions, 'How might Pythagorean mathematics be helpful now? How might a 21st century Pythagorean mathematics manifest now?'

Our present cultural situation is obverse to that of the 6th century BC. The pre-Socratic philosophers were emerging from a kind of unconscious I-*Thou* awareness, a world of relationships with beings, of nature spirits and divinities. Now we live in a predominantly materialist culture, where matter-energy is assumed to be the fundamental reality which we get to know through 3rd person observation of things, an I-*it* consciousness world. This world first came into being unconsciously through early, technological developments[30]. It was the pre-Socratics who made it conscious, beginning the explication of the world of I-*it,* whilst still maintaining connection with the world of !-*Thou.*

This I-*it* separation heralded the possibility for our individual consciousness which only began to manifest fully in Europe in the Renaissance[31] and the 'Enlightenment'. In the centuries which followed there was a slender thread of individuation coming out of the alchemical tradition but the main trend was towards individualism. Adam Curtis' documentary, 'The Century of the Self'[32] demonstrates the extreme of this vividly and shows how, in the 20th century, individualism made people vulnerable to manipulation by business interests, resulting in a loss of their true individuality.

This continues. We live in a global, technocratic plutocracy, a neo-feudal society[33] where corporations and their owners hold the ultimate power. Democracy is a bad joke[34]."None are so hopelessly enslaved as those who falsely believe themselves to be free" as Goethe wrote[35]. To the extent that we

[30] 'The gods and technological consciousness in the ancient, near east', pp11 – 19, 'In the Shadow of the Machine', Jeremy Naydler, publ.2018, Temple Lodge.
[31] This is the key theme of Ernst Cassirer in 'The Individual and the Cosmos in Renaissance Philosophy', transl. Mario Domandi, publ. Univ. of Pennsylvania 1963 (orig'l German ed'n 1927) which followed Jacob Burckhardt's ground breaking work.
[32] https://www.youtube.com/watch?v=eJ3RzGoQC4s&t=47s
[33] Marjorie Kelly, 'The Divine Right of Capital' publ. Berrett-Koehler, 2001.
[34] 'How corporations overthrew democracy' by Matt Kennard and Claire Provost, Bloomsbury Academic, May 2023
[35] J.W.von Goethe, 'Elective Affinities'.

are numbed by our addiction to things, to TV, computer and 'smart' phone screens, we do not have the free, mental, emotional and soul space to discover our true natures. Corporate control of communication media moulds the mob mind into mass formation (as Mathias Desmet calls it[36]) to its own profitable ends.

How has this happened? Martin Buber's explanation is simple. When we are in the world of *it*, we have reduced ourselves to *it*. "All real living is meeting"[37], i.e. living I-*Thou.* This is what has been lost. We can not live continually in this space[38], but when we do not live in this space we do not really live. We can trace one root of our losing touch with our real lives to the Galilean view that the Book of Nature is written in quantitative mathematics.

This belief has given us our highly successful materialist science. It is predicated on Galileo's distinction between what he called primary (i.e. quantitative, mathematical) qualities and secondary (sensory) qualities. In his enthusiasm for what he had discovered through his telescope (wonderful!) he failed to notice that his primary qualities are not sense free, in fact they are determined by the visual sense since all quantitative measure ultimately comes from the physical, visible ruler[39]. And this was only one aspect of vision, the black and white aspect. The beautiful colours[40] of nature that brighten our lives, were relegated to secondary qualities.

Quantitative mathematics is based on vision This is a distance sense[41]. Over the past 500 years we have been increasingly distancing ourselves from our proper reality, the one that belongs to us inherently. We have outsourced our perception of physically inaccessible reality to machines of our making, via mathematical interpretations. The Pythagoreans and still today, shamans and mystics, experience directly realities of worlds of non-physically-perceptible beings.

In fact much or most, of our living is in the realm of the non-physically-perceptible. We just don't think of it that way. Our emotions, for example:

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[36] Mattias Desmet, 'The Psychology of Totalitarianism', transl. Els Vanbrabant, Chelsea Green Publishing, 2022
[37] Martin Buber, 'I and *Thou*', p.17
[38] T.S.Eliot, "Humankind cannot bear very much reality".
[39] Jeremy Naydler, pp. 6-20, 'The Restitution of the Ear' in 'The Future of the Ancient World, Essays on the History of Consciousness', publ. Inner Traditions, 2009.
[40] Anonymous, New York, graffiti artist: "Colour is happiness".
[41] Vision and hearing are distance senses. Touch and taste are contact senses. Smell has aspects of both. The latter three have proven quite resistant to quantitative mathematical classification.

materialist science analyses these in terms of hormones and neuroscience but this is a wholly separate world from our direct experience.

In the 20th century we have been brought up to believe that materialist science is fact and spiritual ideas are beliefs[42]. Note: we believe this. Belief is not part of science. Science is about evidence. Newton famously said :"Hypotheses non fingo", "I don't touch hypotheses". 'Hypotheses' meant statements of belief about reality. Newton wanted to make it clear that his mathematical descriptions, which fitted scientific observations of movements of material bodies, were not statements about reality per se.

This important distinction has been lost. We have scientific theories which are pictorial descriptions of the quantitative, mathematical equations describing experimental results. These are helpful. But they are not descriptions of reality. They give us models that fit equations[43].

A key element of scientific practice and belief is that any theory is only as good as the last experiment confirming it. Scientific theories change. But the fact that they are theories, has sunk from view. The fact that not all scientists agree on interpretations of experimental evidence, is hardly ever given public viewing. Similarly the fact that the kinds of experimental evidence we acquire depends on what theory or model we are positing in the first place, has escaped public notice.

As a result, we are living in a linguistic box of materialist ideology. The practical success of materialist science went to our collective heads and engendered the positivist mindset. This puts man [sic] at the top of the tree of intelligence and all earlier times and understandings as mere precursors to present day knowledge. This perspective tends to denigrate that which it cannot understand. It's time for a fundamental rethink.

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[42] This is a mistaken distinction as Henry Corbin has shown in his writings about Persian mystics and their mutual corroboration of the reality of the spiritual world which they enter.
See C.Jambet, '*La logique des orientaux: Henry Corbin et la science des formes'* publ. Seuil 1983
See also Harald Walach, 'Notitia experimentalis Dei – Experiential Knowledge of God,. Hugh of Balma's Mystical Epistemology of Inner Experience – a Hermeneutic Reconstruction', publ. Analecta Cartusiana, Institut für Anglistik u. Amerikanistik, Universität Salzburg, 2010

[43] The map is not the terrain. The standard model of physics is one description that fits the equations but there are others e.g. Pete Rowlands' nilpotent rewrite model, 'Zero to Infinity', publ. World Scientific 2007 and Nick Thomas' counterspace, projective geometry model, 'Science between Space and Counterspace, Exploring the Significance of Negative Space', publ. New Science Books, Temple Lodge 1999.

This includes re-thinking our views on the 6th century Pythagorean investigation of the nature of number/s. Pythagoras realised that numbers are a different kind of being from material beings, they are ideational beings. Yes, they are very useful in the world of *it* but there is more to them than that. The Pythagoreans observed not only the quantitative aspects of numbers but also their qualities. Some of these we recognise now as mathematical: odd, even, perfect etc. Others such as the relationship between the number 4 and the concept of justice, are alien to us.

We can begin to glimpse a possible meaning for this once we begin to acknowledge the reality and importance of non-physically-perceptible beings and worlds; also to realise that there are different, non-physically perceptible beings and worlds just as there are different physically perceptible beings and worlds[44].

To begin to understand what Pythagoreanism was, and what it could be now, we need to give more thought to these different non-physically-perceptible realities and their interrelationships. Rudolf Steiner's view was that the universe is composed of beings of different levels of consciousness[45] with different kinds of consciousness. This accords with the mostly forgotten, scholastic and Pythagorean research into the essences of things, their being natures, as different manifestations of the Divine. In the 2020 ANPA Proceedings I described mathematical beings as 'imaginary' or 'imaginal'; I am now calling them 'ideational'[46].

Atoms, electrons and other sub-atomic particles, waves, forces, fields etc are ideational beings. We extract quantitative data from our sophisticated scientific equipment and attempt to describe/ interpret reality in terms of intellectual models and theories. This is not a problem per se; it is only a problem when we wrongly accord these theory-based beings more reality than our own direct experience.

The word 'science' itself gradually took over from 'natural philosophy' through the 18th into the 19th century. Natural philosophy still carries the sense of wisdom and relates to nature. It was broader in its area of study and often

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[44] NGG ANPA PROC. 2020, pp.392-4, p.403

[45] Rudolf Steiner, 'Intuitive Thinking as a Spiritual Path', (a.k.a. 'The Philosophy of Freedom', original German, '*Die Philosophie der Freiheit'*), transl. Michael Lipson, publ. Anthroposophic Press N.Y. 1995.

[46] Etymological roots of 'idea': late 14c., "archetype, concept of a thing in the mind of God," from Greek 'idea' "form; the look of a thing; a kind, sort, nature…", in Platonic philosophy, 'an archetype, or pure immaterial pattern'; *oida,* 'I know' in Greek, is the past tense of *idein*, 'to see'. https://www.etymonline.com/

deeper in approach. Isaac Newton was a natural philosopher and his studies were of the natural world, the Bible and alchemy. True alchemists were not concerned with refining material lead to create material gold but rather, as Jung and others have demonstrated, were, like the Pythagoreans, working to refine their souls.

'Science' comes from the Latin root '*scire*', 'to know', from the Proto-Indo-European root, "*skei"*, meaning "to cut, split". This is third person, *it* knowing, the motivation being control of *it*. It is distinct from '*cognoscere'*, whose Proto-Indo-European root is "*gnō".* This meaning of "to know" gives rise to Greek "*gnosis"* and German "*kennen"*[47]. This is in the sense of acquaintance, of *thou* relating, as in the 'King and I' song, "Getting to know you". This accords with the Pythagorean ethos of *philia*, loving friendship, which is clearly needed now.

Let's try to enter a little more deeply into the original, Pythagorean way of thinking and of being in the world. Pythagoras had travelled widely before founding his school with his wife, Theano. He had imbibed the Egyptian Hermetic teaching, as above, so below, or, as without, so within. The inner group of Pythagoreans, the *mathematiki*[48]*,* also adopted the Apollonian practice of incubation[49]. They spent long periods of time, lying in special, darkened places, sometimes underground, in order to receive wisdom, knowings, from the spiritual world. For the Pythagoreans, the *Zeitgeist*, led them away from the old gods of the oracles and initiated them to the Divine Oneness[50], and to a new awareness of numbers as beings.

Incubation was a way to enter the *thou* world, opening awareness to spiritual[51] beings. Today when people in the West practise meditation some practise in this Pythagorean way (without thinking of it as Pythagorean). Others are seeking to change their mental state, to move out of words into more open awareness, to find peace from what William James described as 'the ceaseless frenzy of thinking we should be somewhere else".

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[47] These P-I-E roots are from etymonline.com

[48] The *mathematiki,* those who were accomplished in the wisdom of loving friendship, were the inner group, also called the *esoteriki*, literally 'the inner ones'. Our meaning of 'esoteric' as 'spiritual', 'mystical' derives from the nature of the inner, Pythagorean group. They shared all their possessions. They were all of equal status, men and women. They were vegetarian.

[49] Peter Kingsley, 'In the Dark Places of Wisdom' publ. The Golden Sufi Center 1999, pp. 102-3

[50] The Pythagorean roots are Apollonian. Pythia was the high priestess of the Apollonian oracle at Delphi. Apollo was the sun god, god of Oneness. 'Apollo' etymologically means 'not many', as Iamblichus points out.

[51] It is part of 21st century Pythagorean mathematics to clarify terminology around non-physically-perceptible beings and words, spiritual, ideational, fictional, factional (fact and fiction have very similar etymological roots), mental, intellectual, imaginal, imaginary, ….

Wordless awareness is our primary experience of the awe-inspiring reality of the world in which we live, of which we are part. On this basis we could construct a 21st century Pythagorean epistemology in which the Galilean distinction is reversed. The *it* consciousness is secondary. It comes out of awareness, an I-*Thou* relationship with ineffable Reality.

Read '>' as, 'is greater than, and includes':
Reality > our awareness,
Our awareness > our consciousness,
Our consciousness > our language,
Our language > our reason,
Our reason > our logic[52].

The etymological root of 'epistemology' is Greek, meaning 'to stand on'. Enlightenment reductionist philosophy chooses to stand on logic [53]. This is the desire for external certainty. The Pythagorean way is to base ourselves in our awareness, where we are closest to ineffable reality.

The desire to understand is the obverse of the epistemological quest; it requires us to 'stand under'. In the old Sarum prayer we entreat,
"God be in my head, and in my understanding.
God be in mine eyes, and in my looking.
God be in my heart, and in my thinking……."

We are asking that Godness, the Great Oneness, may stand over us when we are conscious, in our heads; and that Godness may guide us as to where we direct our attention, where we look[54]; and that Godness may help us to open our awareness to allow heart thinking.

When we open our awareness in this way, there is the possibility of a new consciousness. Then we are in a liminal space where heart and head come together, an alchemical furnace of becoming; our hearts open to the vast Oneness and our heads need to change, give up old ideas, allow some unnamed

[52] NGG ANPA Proc.2020, pp.394-5
[53] As A.N.Whitehead wrote "…much nineteenth century philosophy [mechanistic reductionism, logical positivism] excludes itself from relevance to the stubborn facts of daily life", 'Process and Reality', publ. The Free Press 1978, from the Gifford Lectures, 1927-8.
[54] This might be a good aspiration when scientists are considering what experiments they want to make. There is a Sanskrit mantra, '*Padrani karnebi: śrinuyama devań',* 'May my eyes see the good'. This was the conclusion of my paper, 'The Beauty of Mathematics, the Mathematics of Beauty' in ANPA Proc.2020, pp.367-386.

happening to take place in us. Here our 'I-*Thou'*, 'I-*thou'* and 'I-*it*' meet; our identity needs to submit to our integrity.

Our identity is far from our integrity. Etymologically, 'identity' comes from medieval Latin, *'identitatem'*,'sameness'; 'idem' means 'same'. Identities are part of the homogenisation inherent in reductionism. 'Integrity' comes from the Latin 'integer' meaning 'intact, whole, complete', figuratively 'untainted, upright', literally 'untouched'. *Vide* the importance of the integers, the whole numbers.

We establish our identities in language, in our mother tongue. We are social beings. We learn to think in our mother tongue[55]. We learn to say 'I'. Initially we refer to ourselves in the 3rd person (I-*it)*, by our first name. Our first 'I' is an identity constructed from identifying with our 3rd person name.

As we grow up we go on establishing our identities by choosing whom and what we identify with. As 'grown-ups', in order to really grow, we need to be willing to relinquish our homogenised identities and attempt to realise our integrity in our heterogeneous incarnate multiplicity, in our vast, heterogeneous cosmos.

The move from consciousness to language, takes us from our individual reality to a shared world. Languages are our means of communication; they are also our memory holders[56] and our map makers. They exist in written and oral form. The latter is the living medium. Languages are non-physically-perceptible beings. They change, evolve. We can speak of spirits of languages as we do of *Zeitgeiste*, time spirits. We are part of them; they live in us and through us, and our conversations and writings are part of their evolution.

Mathematics is a language and since 1600 AD a symbolic language independent of verbal, vernacular language. It achieved this independence through the arrival of the symbol zero and the subsequent creation of the number line. The philosophical ramifications of this were immense. The great Oneness, the Monas, the ineffable Divine, was no longer the source of number and of all things. It was replaced by zero, an arbitrary point on a homogeneous number line. The integrity of the integers was no longer acknowledged; they were also just more seemingly arbitrary points. [57]

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[55] "*Manas* [mind] is made of words." Peter Harrison, Vedic teacher.

[56] We, human becomings, very seldom have any memories from before we could speak.

[57] NGG, ANPA Proc. 2014, pp.25-52, 'Historical Changes in the Concepts of Number, Mathematics and Number Theory', for more of this story and some of the implications.

At that time the extraordinary possibilities of technical, mathematical advance arising from the appearance of the '*Wunderkind*', 'zero', overrode any ontological concerns or investigation. The Platonic ideals of the Liberal Arts, truth, beauty and the good, were forgotten, rendered obsolete in the rollercoaster ride of the new, bourgeois, 'progressive', industrial, capitalist world emerging…. Pragmatism ruled the day.

And the meaninglessness which this homogenisation of number engendered, was also ignored except by a very few. In the 20th century the integers began to rise in status again, through the importance of the primes in finance for cryptography[58] and the extraordinary relationship between the Riemann hypothesis and quantum theory.

So we now face a mathematics which is seemingly meaningless in human terms: its importance being that it embodies consistent, abstract rules which can be applied in various situations to give results that work[59]. But, as we've seen, this mathematics does not have the certainty that Hilbert hoped for. And, certainly the results have not always been desirable.

So, the fundamental imperative of Pythagoreanism, to develop and work from, the wisdom of love, loving friendship, in the human realm, would clearly be helpful. How might Pythagorean mathematics manifest now in ways that might promote this? Can we work within and on, the mathematics that has burgeoned forth from the 6th century BC seeds, to find meaning in these forms and processes that have emerged since the time when any concern with ontology and meaning was abandoned?

One, clear, Pythagorean task would be to consider the ontological implications of joining these two inherently different entities, numbers and lines. As stated, number was originally studied in two ways, quantitatively, *logistiké*, and qualitatively, *arithmetiké.* The separate study of space, including points, lines, planes and solid forms, was *geometria*. We see here the key, ontological differences between the integers (number) and lines:.

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[58] VI Arnold was somewhat cynical about the driving forces for 20th century mathematics, describing it as "divided into three parts: cryptography (paid for by CIA, KGB and the like), hydrodynamics (supported by manufacturers of atomic submarines) and celestial mechanics (funded by military and other institutions dealing with missiles, such as NASA)". This was only partly tongue in cheek.

[59] This was the meaninglessness which entered mathematics and was pushed under the carpet three hundred years before. When it comes to the surface it is trumpeted as a virtue.

Numbers/ integers	Lines
Whole (indivisible)	Divisible
Discrete	Continuous
Separate, distinct	Meeting, crossing.

Another, important, related, Pythagorean task now is to consider the different ontological natures of one, Oneness and zero, and their possible relationships.

Yet another Pythagorean task is to consider the strange nature of mathematics as language and the philosophical and societal implications of this. The grammar and syntax of mathematical language, are somewhat skew to the parent, verbal language from which it emerged. It's a symbolic language: the written forms are not related to the sounds of the words we use for them. They float free of the living ground of embodied speech.

Proto-Indo-European languages are phonetic. Ancient Egyptian was symbolic, as is Chinese. The symbols of both of these are based in some pictorial meanings. Mathematical symbols (apart from the numerals) have been chosen with some attempt at this but, given the abstraction of the meanings, this is not very apparent. George Lakoff and Rafael E. Núñez, in their *tour de force,* 'Where Mathematics Comes From'[60], analyse mathematical ideas in terms of human, physical experience. But there are still places where the non-physical appears.

The mathematical – verbal separation had not yet occurred at the time when the Liberal Arts flourished in Chartres. Could we envisage a 21st century Liberal Arts where a Pythagorean quadrivium would be given a home, a grounding in the verbal arts of the Trivium? Together the trivium and quadrivium manifest the two principal meanings of *logos* (λογος) from pre-Socratic times. The quadrivium explicates *logos* as 'ratio'. The trivium explicates *logos* as 'word'; this was Heraclitus' domain[61]. This relationship between 'ratio' and 'word' is one way to enter the complex relationship between mathematical language and verbal language[62]. The fact of their being joined in the liberal arts, calls attention to the wholeness they embody together.

[60] George Lakoff and Rafael E.Núñez,'Where mathematics Comes From', publ. Basic Books, Perseus Books 2000
[61] Hermann Diels, '*Die Fragmente der Vorsokratiker'* publ. Rowohl 1957, *Heraklit* , pp.21-31.
[62] Here are a few of the meanings of *logos*: the word by which the inward thought is expressed, the inward thought or reason itself, word (pl. language), saying, proverb, conversation, discussion, report, rumour, story, narrative, prose, speech, ratio, thought, reason, account, consideration, esteem, calculation, reckoning, relation, proportion, analogy…, Liddell and Scott, 'Greek-English Lexicon', publ. Oxford, Clarendon Press 1963.

Naming is a linguistic and mathematical act. A large proportion of mathematical advances (arguably the most significant) have been those of recognising new kinds of connections and naming them[63]. In verbal language naming is paradoxical: it can be labelling and dismissive[64] or it can be invoking and honouring[65]. The former is in the I-*it* world. The latter is in the liminal world, the borderland of I-*it* and I-*thou*.

The fact of drawing together the 4 core areas of technical, Pythagorean mathematics and naming them, the quadrivium, is significant. The technical mathematics that we have now, has no essential definition. It is defined by extension, a wondrously powerful, sprawling web of topics, linked by a symbolic language of formal relationships, separate from verbal languages.

The quadrivium is a meaningful whole of numerical arts. It has a clear subject matter: number (originally, the integers) and its manifestation in space (geometry), in time (music) and in space and time, the universe (astronomy). Number is studied in its quantitative aspect for practical living, and in its qualitative aspect to investigate relationships (*logos,* ratios).

The 7 Liberal Arts were the foundation of civilised education in Europe from Cicero's time. He introduced them in order that citizens should become honourable, responsible members of society. In the 13th century in Chartres cathedral the main emphasis was on spiritual development. The whole design of the cathedral is to this end. The meditative path is written in stone as a labyrinth on the floor of the nave. This has been walked for centuries and is still a key part of pilgrimages to this cathedral of Mary and the risen Christ.

The quadrivium and the trivium both have these dual aspects of material practicality and spiritual meaning. If a 21st century Liberal Arts curriculum were introduced at GCSE level in the UK, might this help to civilise society now? How could a 21st century quadrivium manifest now?

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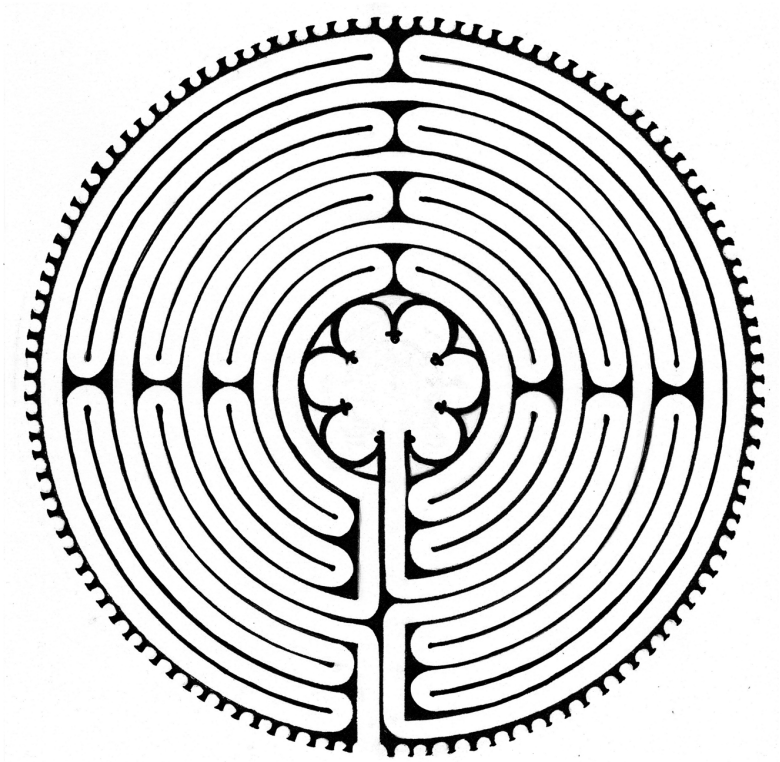
[63] There is something somewhat Rococco in some ways in this activity. Connections are discovered within one area and named and then connections between these connections are discovered and named, and … There is understandable excitement when connections are discovered between different areas of mathematics. This was the case when Andrew Wiles finally proved Fermat's last theorem.

[64] Someone famous once said, "The first time we learn the word, 'bird', it is a wonder. We never see a bird again." We walk through streets of barcodes.

[65] Thich Nhat Hanh, in 1994 UK retreat, "When we say 'orange', we raise it to its highest possibility."
Rainer Maria Rilke, "Perhaps we are *here* in order to say: house, bridge, fountain, gate, pitcher, fruit-tree, window – at most: column, tower… But to say them, you must understand, oh to say them *more* intensely than the Things themselves ever dreamed of existing." The Ninth Elegy, p.57, 'Duino Elegies and The Sonnets to Orpheus' transl. Stephen Mitchell, publ. Vintage 2009, 1st publ.1982, original German publ. 1923.



The stone labyrinth on the floor of the nave of Chartres Cathedral, about 42 feet in diameter. Drawing by Daniel Docherty.

https://www.sacredartofgeometry.com/

Number, in the 13th century meant the integers, the whole numbers, which begin in Oneness, the source and container of all. All numbers are ratios of one[66]; the number (integer) beings speak of relationships. Twoness can be opposites and/or complements. *Vide* the famous yin-yang circular symbol. Threeness arises in one way as the two elements with the relationship as the third. Threeness is different from the quantitative three composed of three homogeneous elements. Threeness has a geometrical embodiment in the triangular form which is both active, having direction, and stable in its material form[67]. And the process goes on but not in a strict 1,2,3,4,5,… way. Two and three can lead to the interlinking triangles, the Star of David, and the hexagon, the lily and the bees. We live in a never ending, wondrous web of integral interrelationships. It can be helpful to add '-ness' to the name of the number when we are talking about its qualitative aspect, leaving it bare for its quantitative aspect.

[66] Prof. John O'Connor of Mactutor, St.Andrews, on BBC radio 4, 'In our time', on Pythagoras, 10th December 2009

[67] Triangles are for that reason the basis of Buckminster Fuller's famous structures.

Geometry, etymologically, is the measure of earth, the laws of material space. This is the mathematics of the visual sense. The geometrical beings speak of meetings: lines cross in points; planes meet in lines.

Music, etymologically, is of the muse (Greek, *mousa*). It is the mathematics of our hearing. Music calls us into now, the only time we have[68]. Our senses of hearing and seeing have different relations with time. Our eyes see stability in material space and notice a differentiation when there is movement. We have the obverse relationship with time through hearing. Sound is momentary and it is only through music that duration appears. The music of harmony and melody and the music of speech, create meaningful duration[69]. Music also invites us to move physically, to dance and sing, and to move in our minds and souls and spirits.

Astronomy, etymologically, is the law of the stars (Greek, *astra*, stars; *nomos*, law). The astronomical beings call us to our relationship with the cosmos, to sense our place (geometry) in the harmony of the spheres (music).

In a present day trivium, grammar would supply the rules of our native tongue, the linguistic box we grew up in. In 13th century Chartres it was the grammar of Latin, the academic lingua franca. Logic gives us the basis for reasoning. Rhetoric instructs us in how we can use language creatively, meaningfully and beautifully.

The Quadrivium now

'Number' is no longer only the positive integers, but also the negatives, the rationals, irrationals, transcendentals, imaginaries, complex numbers – so many, new, different kinds of number being to meet. We have also discovered more about the integers themselves, both as a group (prime numbers and the Riemann hypothesis) and individually.

These heterogeneous entities of plurality, are all processes, ratios of one, relationships with oneness. They speak not only of the physical world around us but also of the many aspects of our individual psychological, spiritual wholeness, our integrity, oneness.

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[68] Incredible String Band, "This moment is different from any before it. This moment is different; it's now. And if I don't kiss you, that kiss won't be tasted. We'll never, no, never get it back. But why should I want to. I'll be in the next moment. Sweet moment. Sweet lover. Sweet now." Album, 'The Hangman's Beautiful Daughter, 1968. Also, Eckhart Tolle, "The Power of Now" publ. New World Library, 1999.

[69] Gaston Bachelard, '*La Dialectique de la Durée'*, publ. Presses Universitaires, 1950

To appreciate them qualitatively more deeply we can begin in a Goethean phenomenological way by collecting information about them, information about their different manifestations both in standard mathematics and in life. In this vein Ernst Bindel's '*Die gesistigen Grundlagen der Zahlen'*[70] goes into considerable detail, particularly concerning the first twelve integers.

We can also inquire as to the meaning of archetypal occurrences of integers. Modern mathematics has neglected to look at these, considering them to be mere coincidence. One example is the 3,4,5 Pythagorean triangle. The fact that the first three[71] consecutive integers create orthogonality, a right angle in space, linking number and space, is wondrous.

How might we approach the meaning of the fact that there are five and only five perfect 'solids' (actually, forms) in space? This is clearly a significant, archetypal occurrence. And that they embody polarity of point and plane around line? The tetrahedron, is self-dual (it has 4 points and 4 planes); the others are two pairs of duals. The cube has 6 planes and 8 points; the octahedron has 6 points and 8 planes (6 = 2x3; 8=2x.4). The dodecahedron has 12 planes and 20 points; the icosahedron has 20 planes and 12 points (12=3x4; 20=4x5).

And there are two streams. The 3, 4, 5 (again) stream, the tetrahedron, cube and dodecahedron have respectively 3-sided, 4-sided and 5-sided faces. The triangle stream, tetrahedron, octahedron, icosahedron, all have equilateral triangle faces.

Form	Points	Lines	Planes	Face edges
Tetrahedron	4	6	4	3
Cube	8	12	6	4
Octahedron	6	12	8	3
Icosahedron	12	30	20	3
Dodecahedron	20	30	12	5

In the physical universe as well, there is mystery in the appearance of specific numbers. Martin Rees' 'Just Six Numbers' lists six fundamental, physical constants such that, if any were different, our universe could not exist as we

[70] This is, at last, in the process of being translated into English.

[71] In the view of most Greek mathematicians number begins with two, but some held that two was not a number as such but the principle of evenness and the unlimited (*apeiron)* and as such number begins with three.

know it[72]. The fact that there are a number of dimensionless, physical constants caused Sir Arthur Eddington and later various ANPA physicists and mathematicians[73] to investigate ways that all physics might be derived from pure mathematics.

And, of course, we have the task of uncovering possible meanings of the 'new', non-integer numbers. This investigation follows that of investigating the one-oneness-zero, interrelationships and the polarities involved in the creation of the number-line.

Geometry has metamorphosed deeply since Pythagoras' time. This was the geometry later codified by Euclid in his 'Elements', where parallel lines never meet (the parallel postulate), believed to be the space of our world. In the 17th century this was transformed into a quantitative form, Cartesian geometry. This worked well and continues to do so, for many branches of science and technology. But two centuries later, through the deeper questioning of pure mathematicians, various non-Euclidean geometries were discovered. These led into topological ideas independent of quantitative, visual measure.

In fact, a geometry without quantitative measure, had already appeared in the 17th century: projective geometry. And interestingly Descartes and Desargues (whose theorem is archetypal in projective geometry) were mathematical friends. Also, ironically, it evolved from the 15th century need for accurate architectural measurement. This was achieved through the invention of perspective drawing by the Italian architects, Brunelleschi and Alberti, which led to projective geometry.

This is a geometry where the elements are the points, lines, planes and solids of Pythagorean *geometria* without quantitative measure and where parallel lines do meet at a point at infinity. The crossing of 2 lines and their meeting in one and only one, point, is the fundamental phenomenon of projective geometry. It unfolds with more, different crossings and their resultant forms and projections into extraordinary, beautiful relationships and integer phenomena.

The world of I-*thou* concerns meetings. Might there be ways that these correspond to meetings in projective geometry? In the world of I-*Thou,* meetings change us. In materialist science, in the I-*it world,* people conduct

[72] Martin Rees, 'Just Six Numbers', publ. Phoenix, Orion books, 1999

[73] It was one particular model, namely the Combinatorial Hierarchy (CH) which was the catalyst for the founding of the Alternative Natural Philosophy Association (ANPA) in 1980. See Clive Kilmister, 'Scientific Essays in Honor of Pierre Noyes', Series on Knots and Everything Vol.54, pp. 203-209, publ. World Scientific 2014

experiments on things with a view to obtaining repeatable results. But the I-*thou* world is different. Even if we think of experiencing events, these events still have a thing-like nature. It is only when we encounter, meet others, Other, in such a way that something happens in us, to us, with us, that is beyond our norm, that we are freed into I-*thou.*

Real meetings change us. When we are truly alive we meet moments. Our real living is made up of such moments. These are moments of awareness beyond consciousness, beyond language. From moments of awareness, meaning enters consciousness with momentum. These are the 'Aha!' moments. Personally I have experienced such moments in the act of drawing projective geometrical forms. This is key. It is not the intellectual knowing of the theorems, it is the living engagement with them, that is the true Pythagorean telos.

We also have the tasks of investigating human and spiritual meaning of other non-Euclidean geometries, topology, knot theory and ideas that have evolved from these.

Music and Measure

Musical measure is the natural measure of our living. . It is the living measure of our relaxed breathing when we allow the breath to breathe us. It is the measure of our blood, of our heart beats. A healthy heart does not beat in a regular, clock-like rhythm. When that happens it is a sign that a heart attack is imminent. Homogeneous, clock time is an imposition of visual, spatial measure on natural, moment-ous life[74].

In written music there are bar lines indicating a basic time period and the notes have lengths, doubling the length each time: semiquaver to quaver, quaver to crotchet etc., up to semibreve. But these are not fixed time lengths. The tempo of any section of music is given, from slow, andante, to fast, allegro.

These are not metronome, homogeneous time signatures. . It is the interpretation of the musicians, according to how they feel a piece calls to be played, which determines the timing. Number is significant in music not according to quantitative time but in integer relationships. The time signatures are integer ratios, for example, 3:4 or 5:8 etc. The characteristic 3:4 of the waltz

[74] Rainer Maria Rilke, "*Heil dem Geist, der uns verbinden mag; denn wir leben wahrhaft in Figuren. Und mit kleinen Schritten gehn die Uhren neben unserm eigentlichen Tag…",* "Bless the spirit who loves to link us; for we live truly in shapes, and the clocks in little steps walk alongside our actual day(s)."

feels totally different from 4:4 march music. Harmony and melody also arise through integer relationships between the tones, 2:1 for the octave, 3:2 for the perfect fifth, 4:3 for the fourth.

One Pythagorean project is to teach mathematics in schools through music. Mathematics teaching has become increasingly dependent on computers, increasingly screen based, increasingly individually separate. To teach through music would involve playing music with others. This would encourage listening, *philia*, cooperation and collaboration.

Of course, both music and geometry involve both integer and number-line phenomena. These are all to be researched in a Pythagorean programme.

Astronomology

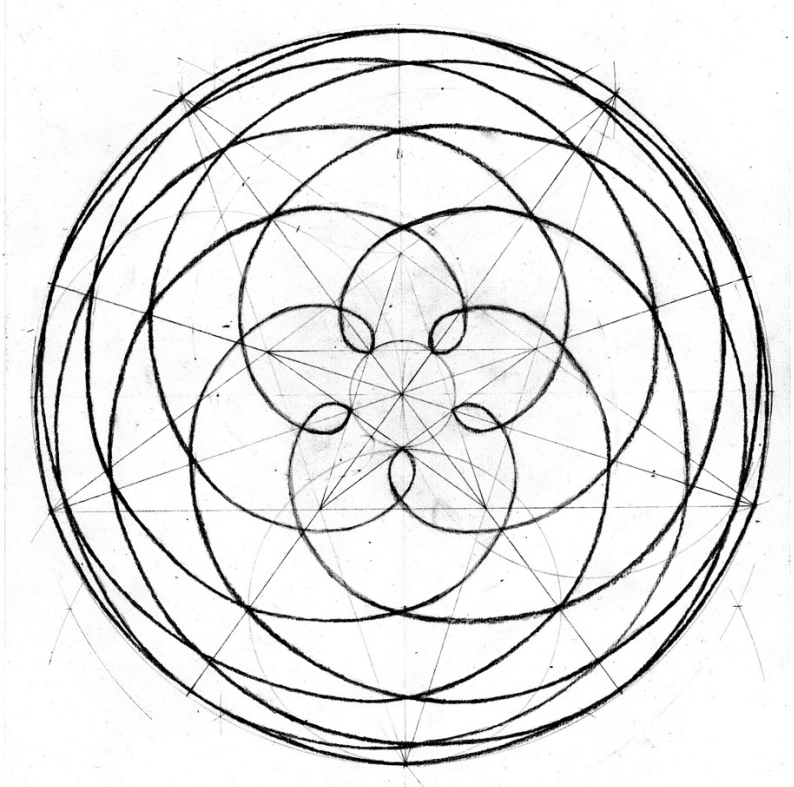
Astronomy now has advanced physically beyond what might have been envisaged in Pythagorean times. But how are we connecting with the cosmos?[75] The Natural History Museum states that there are about 2,000 active satellites orbiting Earth at the moment and 3,000 dead ones littering space. There are also around 34,000 pieces of space junk bigger than 10 centimetres in size and millions of smaller pieces that could cause damage. Elon Musk now has 800 satellites in orbit and plans to have 12,000 by 2026 as part of his Starlink project. China plans to launch 13,000 satellites in response. We see again the lack of the precautionary principle and lack of respect for the cosmos.

Astrology has traditionally interpreted data about planetary movements with respect to each other and to the constellations, in terms of what these might mean for individual humans and/or humanity. When we consider the universe to be composed of beings with different levels of consciousness, then we are faced with other questions about the being natures, not just of Gaia, our Mother Earth, but also of the other great beings of our solar system, and the myriad great beings beyond.

These are enormous questions. They involve not only the vastness of space but also astronomical time periods, millennia and much more. Hartmut Warm has used the possibilities of present day technology not to send more bits of metal

[75] "Where is the wisdom we have lost in knowledge? Where is the knowledge we have lost in information?", T.S.Eliot, 'The Rock' 1934

into space, rather to use the potential of high speed computing to discover beautiful patterns of interplanetary relationships over millennia[76].



Venus-Earth cycle. Drawing by Daniel Docherty[77]. https://www.sacredartofgeometry.com/

Astronomology involves number, geometry and music. We need to find our natural measure to create the integral music of our individual living, which needs to be in tune with the cosmic measure in the harmony of the spheres.

The quadrivium brings wholeness to mathematics. Wholeness involves heterogeneity.

Geometry Space Vision Visual measure Number line

[76] Hartmut Warm, 'Signature of the Celestial Spheres', transl. J.Collis, publ. Sophia Books, Rudolf Steiner Press 2010

[77] When plotted geocentrically, from an earth-centred perspective, there is a rhythm in Venus' motion. After 8years, it returns to the same place in our sky on about the same date. This is known as the 8-year cycle of Venus, and stems from the fact that 13 Venusian orbits (13 x 224.8 days) very nearly equals 8 earth years. This cycle was known to ancient peoples such as the Mayans. Today, many know it as the pentagram or petals of Venus. This is because, over the 8 years, each phenomenon, each relative position of Earth, Venus, and the sun, occurs 5 times. Then, over the next 5 years, they repeat 5 times almost identically.

Music	Time	Hearing	Musical measure	Integers

Once we recognise the inherent difference between the modalities of space and time and the correspondences between space and vision, and between time and hearing, then we can begin to understand the deep wisdom of the quadrivium.

Geometry and music are the sacred arts that reveal the beauty we are blessed to discover and create, through these two exquisite senses. We can begin to appreciate the different natures of the measures of each. The continuity of space penetrates infinitesimal detail[78], and extends out into infinite distances. Ratio in musical rhythm and harmony moves our bodies, hearts, souls and spirits.

We observe here another apparent difference in the mathematical natures of seeing and hearing. We can map the twofoldness of vision directly onto the number-line: infinite extension and infinitesimal nearness. Mapping the twofoldness of hearing encompasses ratio as rhythm in temporal duration but in harmony we enter another whole, different world in an instant. This is only an apparent difference in that when, in materialist science, visual measure of infinitesimal detail has been attempted, the vastly different quantum world was opened!

So, might there be any meaning in such a correspondence for number and/or astronomology?

Number	?Integrity	?Spiritual sense	Proper(own) living measure
Astronomy	?	?	?

It could be that the latter two are incommensurate with this template which is, in any case, limited. The senses of touch, taste and smell are more physical and not part of the quadrivium. All five physical senses[79] have elements which are beyond the reach of number, quantitative and qualitative, the integers and the

[78] William Blake, ".. a world in a grain of sand, Heaven in a wild flower..", 'Auguris of Innocence'

[79] When we look outside our western mathematical heritage to the Hindu Vedas, the original root of the Hindu-Arabic numerals, there is a very different meaning to measure. In this tradition all five physical senses have their own, individual measures. I gave a presentation on this at the 2019 ANPA conference. The paper will be completed for the next Proceedings.

number line. All the senses[80] are involved individually and together in various ways when we begin to move from Goethean phenomenology into alchemy. This is when we move into the I-*thou* world, opening to non-physically-perceptible beings. This is definitely work in progress.

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Let's begin to open ontological questions of oneness, one and zero, albeit briefly. Oneness, the Monas, is the great wholeness, of which we are part. This is the ineffable Divine, recognised in various aspects in different religions as the supreme Godness, God, Allah, Jehovah, Brahman, …

For the 6th century Pythagoreans the Monas was also the source of number which unfolded in the tetraktys, as mentioned. We could now also interpret the tetraktys as a tetrahedron, the 4 points (or planes) of which could be Divine harmony, truth, love and beauty[81].

In my ANPA paper, 'The Beauty of Mathematics, the Mathematics of Beauty'[82] I showed how the neuroscientific work of Prof. Zeki and University College London led to the conclusion that looking for ways to define beauty externally is to miss the point of beauty entirely. Appreciation of beauty is part of being human. Developing our sense of beauty is part of growing our humanity. In this way it is a (Pythagorean) mathematical telos to help us grow our humanity by helping us develop our sense of beauty.

Appreciation of the great Oneness of which we are part, can be part of this endeavour. The great Oneness is the whole. One is the individual, a wholeness within the great Oneness. As individual human becomings, we sense the wholeness of our spirit and we seek to live our integrity in the heterogeneity of incarnation. One is also the unit, the basis of quantitative measure: one foot, one centimetre, one ounce, one gram etc.

Zero is the starting point on a number-line. It is 'no – something'. It began as a record of an empty place in abacus calculations of exchange of goods and money. In this context it only has meaning with respect to measurement of something. It emerged from its original home as part of trading, and entered mathematics fully as a key element of the birth of symbolic algebra, in the 16th century.

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[80] Materialist science has now identified many more physical senses and Rudolf Steiner wrote of 12 senses, not all physical.

[81] In the Platonic trinity of beauty, truth and the good, harmony and love are subsumed in the good.

[82] NGG ANPA Proc.2020, pp.367-386

This created mathematical mayhem which surfaced clearly in the subsequent development of the differential calculus with the question as to what happens when you divide a non-zero number by zero?[83] It is even more problematic to consider what happens when you divide zero by zero.

From a Pythagorean, ontological point of view, when zero floats free from being zero apples, plums, pears, pounds, dollars, feet, Newtons etc., i.e. a dimensional grounding, its 'natural' home is as a point. This could give rise to an ontological picture where Oneness is the great whole beyond time and space, zero is the point-instant, now.

Oneness is no-thing-ness, the great whole before the opening into differentiation, before space and time. With zero as the point-instant now, time, past and future, immediately come into being. One as unit can be the beginning of a dividing of time, and/or a pulse in a rhythm of time. One as individual (indivisible) is the first integer (indivisible), to which all the unfolding integers relate as ratios, rhythms and patterns. It can also represent our human, individual (non-negotiable[84]) spirit.

This is a very brief introduction to possibilities of 21st century Pythagorean mathematics where our underlying telos is work on ourselves. All knowledge is human knowledge. I hope I have shown how, once we acknowledge the being natures of this world[85], this telos also opens ways to broaden and deepen our knowing of the world we inhabit.

I want to bring in a quote from Paul Dirac, for three reasons. Firstly his beautiful, seminal book, 'Principles of Quantum Mechanics' is significantly Brechtian. Brecht, like Shakespeare and Grotowski, did not advocate pretence; they wanted the audience to be clear that what they were being shown, was a show. The magic of theatre, physics and mathematics is a matter of wonder not pretence. Euclid similarly presented his *koina*, 'common notions' at the beginning of 'The Elements'. Dirac followed Euclid's template which came from the Pythagorean *deiknymi*, 'I show', which was the original term for

[83] Bishop Berkeley's criticism of differential calculus in 'The Analyst', 1734, ("ghosts of departed quantities"), "Absurdities due to division by zero: An historical note", The Mathematics Teacher, 1929, 22(6):

[84] Beyond the reach of the ever dividing number-line of trade negotiations.

[85] Barbara Mclintock, who was awarded the Nobel Prize in 1983 was clear about her relationship with plant beings, "It might seem unfair to reward a person for having so much pleasure over the years, asking the maize plant to solve specific problems and then watching its responses.

mathematical ‘demonstration’, which in turn became called ‘proof’ as part of the growing culture of the expert and often lost its demonstrative power[86].

Secondly his relativistic wave equation in quantum mechanics introduces angular momentum as the key element whereas Einstein’s more famous equation of special relativity has energy as the key element. Momentum is a vector; it has direction. Energy is a scalar; it lacks direction. Meaning enters awareness with momentum into consciousness. Awareness lives in the quantum pre-consciousness world, Bohm’s ‘implicate order’. Dirac’s equation is an analogue in materialist science of the human phenomenological moment of realisation. This calls excitingly for deeper investigation.

Thirdly he was Bristolian[87], like Cary Grant[88], Banksy[89], Tansy Davies[90] and me[91]. So “It is more important to have beauty in one’s equations than to have them fit experiment.”[92] Beauty takes us beyond materialist scientific experimentation – out of the *it* world into *Thou* .

And the final utterances rest with the birds (“the arrows of the wise”[93])

“If the birds and the books disagree, listen to the birds.”[94].

[86] NGG, ANPA Proc.2020, pp.388-400. See also Imre Lakatoś, ‘Proofs and Refutations’, British Journal for the Philosophy of Science, XIV, publ. Cambridge Univ.Pr. 1976
[87] https://www.britannica.com/biography/Paul-Dirac
[88] https://www.britannica.com/biography/Cary-Grant
[89] https://banksy.co.uk/out.html
[90] https://tansydavies.com/biography/
[91] https://www.livingspiritmathematics.co.uk/
[92] https://www.theguardian.com/science/2016/nov/21/magic-numbers-can-maths-equations-be-beautiful
[93] Incredible String Band, ‘Job’s Tears’ on ‘Wee Tam and the Big Huge’, https://www.youtube.com/watch?v=Dd5yq76q51c
[94] Saying told to me by Inari Alanko, part Finnish, part Sámi, in The Truth and Reconciliation Commission of the Sámi People. Possibly an old Finnish saying, possibly from John James Audubon, ornithologist and bird painter, https://www.nhm.ac.uk/discover/john-james-audubon-and-the-birds-of-america.html.