**Pythagorean Philosophical Mathematics – a 21st century possibility (devoutly to be wished)**

Content of the Pythagorean teaching

The core of Pythagorean teaching is number.  But their concept of number was deeply different from our modern concept, which only began to come into being at the end of the middle ages in Europe. In fact, it was this expansion of the concept of number that signalled the end of what we call the middle ages. It was this seemingly small change that made all the subsequent, technological and economic, material advances possible.

We shall see how this came about and we shall also see what was lost in the process.

Firstly, in order to get some idea of what was gained and what was lost, let us attempt to enter the earlier world of Pythagorean number.

The word for number then was *arithmos.* These are the whole numbers, also known as the integers or natural numbers. One is not a number. It is oneness, the great oneness that contains all. In this sense, one is the biggest number. Although this sounds strange to us, it is actually quite reasonable; multiplicity only begins with two!

*Arithmetike* was the study of the qualities of numbers. The Greek word was *eide*, one meaning of which is ‘forms’. These qualities included both what we nowadays consider mathematical, for example, being odd or even, triangular, square etc and what we call numerological, for example, qualities of justice, male or female etc. Let us not immediately dismiss the latter. We shall investigate them later.

*Logistike* was the complementary study of number, number as quantity; the Greek word for this, was *hule.* This was the practical study of calculating.  *‘Logos’* means ‘ratio’; it also means ‘word’ or ‘reason’.

Geometry was distinct from *arithmetike* and *logistike* not only because adding and subtracting, multiplying and dividing are different from drawing shapes, as we might think of it now, but for a deeper reason. Lines were recognised as having a different quality of being from numbers.

Number, meaning the whole numbers, are discrete, can not be divided, as they would then no longer be whole. Lines are continuous and infinitely divisible.

The study of geometry was the study of space, consisting of points, lines, planes and solids.

The 7 liberal arts

There are many stories to tell about what happened when the Pythagorean school was broken. I shall return to these later.

Now we shall look at the continuation of the Pythagorean teaching in the quadrivium of the seven liberal arts. These reached their high point in the school of Chartres cathedral.

 In Cicero’s time they were taught as the basic educational curriculum for a citizen. In Chartres they were again given their true position, as an amplification of Pythagorean principles, providing a way to ascend to communion with the Divine.

*(Photo of stone carving of Pythagoras on the Western façade of Chartres cathedral)*

The foundation curriculum was the trivium, the three verbal arts, grammar, logic, rhetoric.  Following this study, students could progress to the four mathematical arts, namely:

Arithmetike the study of the qualities of the integers.;

Geometry, Number in space, (at that time Euclidean geometry); Music, number in time;

Astronomy, number in space and time.

The Hindu-Arabic Numeral System enters Europe

As is often the case in history, whilst the seven liberal arts had risen to new heights in Chartres, in neighbouring Italy Fibonacci was introducing and popularizing, a symbol which would bring about the marginalisation of the liberal arts for centuries.

This was the symbol, or sign, for zero. *[sign and symbol are in fact importantly different – endnote]* This is key to the Hindu-Arabic numeral system whose introduction revolutionised calculation and changed the European world..

Up until that time in Europe, calculation was done with the Roman numeral system. This does not have a zero, and as such, it is also not a numeral place system. The decimal numeral system that we are all used to today has 10 symbols, 0, 1,2,3,….,8,9 and we differentiate more numbers by assigning meaning to the places where the numerals are written. We start with the units place, then th 10’s place, then the 100’s place, then the 1000’s place, going on as far as anyone wishes. As children lots of us have written numbers with lots and lots of noughts just because it’s so amazing that we can!

*[endnote to Robert Graves’ poem]*

In the Roman system there were individual signs for some of the numbers, starting with I for one, V for five, X for ten, L for 50, C for a hundred, D for 500, M for 1,000. Note that they did not need more.  The other numbers were constructed according to certain patterns of these signs. For example, three is written III, four is written IV, 12 is written XII, 24 is written XXIV, 87 is written LXXXVII, et cetera.

Imagine how difficult it was to calculate in this number system. Try multiplying 7 x 16 as VII times XVI. Monks spent weeks calculating the date for Easter (a movable feast) each year.

The Hindu-Arabic numerals, the basis of our present-day number system, entered Europe through trading. Whilst the Crusades exhausted the resources both of the feudal lords and the Church, they engendered the emergence of new economic, social, cultural and political forms.

Key to all of these was the Hindu-Arabic numeral decimal place system, which was initially a record of calculations with an abacus. The zero, initially, was simply a record of the empty place on the abacus. Fibonacci’s book was called ‘Liber Abaci’ (‘the book of the abacus’). And key to this was the sign (and symbol) for zero.

*(Photo of a real abacus & explanatory drawing)*

The deep, ontological difference between the notion of number for Pythagoras and that which entered with the Hindu-Arabic numerals, is that the latter are infinitely divisible. The identification with line was irresistible and powerful. These days we have all grown up with the idea of number as a line, one dimension, starting from zero.

Graphs are, for us, the obvious way to show he quantitative relationship between different things. They are Descartes’ invention in the (??) century and could only be born as the result of the new, Renaissance mindset and Hindu-Arabic numerals, which had influenced this.

For us the notion of number as an infinitely extendible, infinitely divisible line, seems like common sense. Certainly it is now something we have in common worldwide. And indeed it has filtered into our senses. *(Footnote – see section on mathematics of the senses)* For example, we see space as being 3-dimensional according to this Cartesian view.

This is the case even though from the end of the 19th century mathematicians had shown that there were other kinds of space (*Riemannian, Lobachevsky, projective – endnote).* Indeed, Einstein’s relativistic space, now used in the GPS which supports satnavs etc has not yet entered our way of seeing. It is ‘far out’, the subject of science fiction, such as ‘2001 – a Space Odyssey’.

3-D is, in fact, not natural, any more than clock time is. And clock time would also not be possible without the notion of number as an infinitely extendible, infinitely divisible line.

Mathematicians were aware that something had been lost. The loss was of a holistic view of life, of human life within the world, within the universe. The Pythagorean notion of number was part of a holistic cosmology. ‘Cosmos’ is also a Pythagorean word: it combines beauty and order.

Although they were aware of the loss, the momentum of the wealth of new possibilities overrode all the seeming quibbles in the way. And in the practical world of business and industry, philosophical doubts had no place, were ignored.

A new, fragmented world view: progress and pragmatism

This new number system was part of the new European world that came out of the Renaissance and

The new, emerging world view, with its promise of progress, was fragmented. Descartes’ infamous mind/ body dualism (date?) still hangs over us today. (more to say here) The tide only began to turn at the end of the 19th century.

So, back to numbers at the end of the middle ages. The new ease of calculation created many new mathematical possibilities. The whole concept of number was expanding, (footnote – rationals, negatives, roots etc) but with the lack of a holistic framework, this was a pragmatic expansion. The Italian mathematician and gambler, Cardano, (date? Ref. my doctorate + Ian Stewart) spoke for many when he said that he did not know what these new numbers were, but he used them because they worked.

We see here the beginning of the idea that to understand reality is to understand the workings of things. This is fundamentally different from the Pythagorean view of understanding (more a standing under) which was to understand the different. essential natures of things or beings (the *ousia*? Or is this substance?). This is based on the understanding that all beings are part of Divine Oneness.

The subtext of the question, ‘How do things work?’, is ‘How can we make things work for us?’ This was encapsulated in Francis Bacon’s often quoted (somewhat inaccurately), ‘Knowledge is power’. His intentions (like so many that pave the way to Hell) were good. He believed that the new, technological science (‘scientia’) would improve the lives of all people.

This belief has continued to wreak havoc for centuries afterwards. It rests on the basic mistake that improvement is merely material improvement. The new science lacked the acknowledgement of the great gift of life in this amazingly complex and beautiful world. This is gross ingratitude and hubris replaced humility in too many hearts and minds (mostly male).

(Quote, Shakespeare, Laseu, All’s well that ends well)

‘Hubris’ is the right word here. It is little known today. Is this because we swim in it? Because we live in it? Materialism is a hubristic, belief system. It proposes that everything emerges randomly, mindlessly and places human minds at the pinnacle of randomness.

Towards the end of the 19th century problems began to show up in the foundations of the new, purely abstract, technical, mathematics, also in the mechanistic science based on this almost totally quantitative mathematics.

Moving into the 20th century, new mathematical disciplines arose, non-Euclidean geometries, non-Aristotelian logics, topology, catastrophe theory, chaos theory, complexity theory. In subatomic physics quantum theory overthrew the idea of the ideal, Newtonian observer. Observation itself was seen to be an intervention. Complementarity and uncertainty were part of this new theory.

As is normal in human behavior, most of the professionals attempted pretty successfully to contain these radical developments within their own subject areas and refused to look at the far-reaching implications. (ref to Thomas S.Kuhn) There were notable exceptions, for example, Schroedinger and David Bohm.

My critique does not deny that this one-sided, quantitative, mechanistic mathematics has proven immensely successful in its own way, giving us extraordinary technological advances. But its negative side effects have become more and more apparent. It Is time to revisit the original, Pythagorean wholeness.

These complementaries are:

Not only quantitative analysis but also qualitative.

Not only analysis, but also synthesis,

Not only head-based, but also heart-based and body-based thinking,

Not only visual, but also acoustic, musical mathematics,

Not only technical, but also whole person, spiritual mathematics.

It is time to move from the domination of quantitative number towards heterogeneity of life of the senses, for example, and research what mathematics might reveal itself here.

Here are some beginnings:

Number: Francoise Chatelin and creativity

Number in Space: George Adams and projective geometry and tolology

Number in Time: Jazz, 12 tone, microtone and serial music, and overtones

Number in Time: Astrosophy

And more… Paul Schatz, John Wilkes, flowforms

The keystone for Pythagorean research and understanding (then and now) is that we, human becomings, have been given the great privilege of participating in the life of this awe-inspiring universe.

As Pythagoreans we realise our responsibility for all our actions. Our attention is an action, also our intention

As incarnate beings, we impact physically with the other beings with whom we share this universe, human, animal, plant, mineral, seen and unseen.

Different life forms have different modes of communication, of intelligence, of sentience, of time scale.

We need to learn how to converse in different ways with different beings.

We can see the Vesica Piscis as a 2-d image of 2 spheres interpenetrating such that the centre of one is on the circumference of the other.

The form of the intersecting space can be seen as a highly convex lens. We can imagine 3 Cartesian, mutually rectilinear lines passing through the centre of the lens.

The lens could be a lens of now ((((HAWK CAME – 2.14 pm 17 Oct 2019 Polopos, Andalucia))

The 3 lines are the 6 directions, up, down, left, right, forwards, backwards – also past and future.

Attending, meeting, perceiving, conceiving.

The heterogeneity of life.

Mathematics of the senses

The point centre is the now where we decide. The seed in the infinitude of the human heart.

Modern algebra and differential calculus were born.

C.S.Lewis and T.S.Eliot quotes

21st century Pythagorean mathematics embraces change, process as fundamental, a la A.N. Whitehead.

ALL NUMBERS ARE RATIOS OF/ WITH ONENESS

19 to 20… 1st to 2nd to 3rd millennium: now time to embrace paradox and contradiction in synthesis – humanity to develop this ability in this millennium