TOWARDS A 21st CENTURY PYTHAGOREAN MATHEMATICS : A BRIEF INTRODUCTION

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In order to explain the importance now of 21st century, Pythagorean mathematics, I first need to say something about the original, Pythagorean mathematics of 6th century BC Greece. This was the very beginning of mathematics in Europe. The word itself was coined by Pythagoras. The Greek *'ta mathemata'* means 'those things which have been learned'. This is not a dry, technical, abstract learning. Certainly, this was part of it but it was much more. It was a holistic learning.

In his teaching Pythagoras brought together knowledge that he had acquired from his visits to Egypt and middle Eastern countries[1], and added his own discoveries. The Egyptian, hermetic saying, 'As above, so below' or 'as without, so within', was at the heart of the Pythagorean philosophy. This insight, that the structures and processes we see in the external world, tell us about our own internal, mental and psychological structures and processes, is the basis of true alchemy. This was maligned and marginalised from the 18th century on[2], when chemistry, its younger sister, became more powerful. In the 20th century Jung resurrected it and through his wide-ranging psychoanalytic work, confirmed the truth of the hermetic insight. To the extent that we do not recognise this,

[1]Peter Kingsley, an eminent, ancient Greek scholar, in his book, 'A story waiting to pierce you', tells, and evidences, a story of Pythagoras and his travels, radically different from modern, academic versions. This is a story of Pythagoras as a shaman and more, with a message of vital relevance today.

[2]The fact that there were tricksters who called themselves alchemists does not disprove the value of true alchemy. There are tricksters in all walks of life.

we live our lives half asleep[3]. As Socrates said, "The unexamined life is not worth living".

The word 'philosophy' also comes from Pythagoras[4]. Its original meaning is not 'love of wisdom' but rather, 'wisdom of love'. *'Philia'* in Greek means 'love' or 'friendship'; *'sophia'* means 'wisdom'. They are joined in a similar way to other Greek, compound words such as 'geography' (*ge*, 'earth'; *graphein*, 'write', 'description'), 'geometry' (*ge*, 'earth'; *metrein*, 'measuring'),
'theology' (*theos*, 'god': *logos*, 'study') 'astronomy' (*astra*, 'stars'; *nomos*, 'law'), et al. Love, in the sense of friendship, was the foundation stone of the Pythagorean community. The inner group, the *mathematikoi*, shared all their worldly goods and their reverence for the natural world was evident in their vegetarianism.

To love someone or something engenders the desire to get to know the other better. The discoveries of the harmonious, numerical relationships that exist in space, in music, in the heavens, all come from loving attention. It is from Pythagoras that we have our knowledge of the relationship between the sides of a right angled triangle (the archetype being the 3,4,5 triangle), also of musical ratios, 1 to 2 for the octave, 2 to 3 for the perfect 5th, 3 to 4 for the 4th, et cetera.

As well as investigating these relationships between numbers, Pythagoras introduced the new idea of mathematical demonstration, i.e. a sequential relationship of statements such that, if one statement is true, this implies the next one is true etc. This idea is also essentially a creation of love. It shows the desire to share with others how one might come to understand something. It also shows respect towards the person receiving the explanation: it offers the other an opportunity to question the deductions. This was radically different from the situation in Egypt where mathematics was in the sole hands of the priesthood.

It is also different from the situation now where there is again a knowledge power structure embedded in our culture, similar to

[3]This is a key theme of Arthur Koestler's 'The Sleepwalkers'

[4]They [the Pythagoreans] were the first to call themselves 'philosophers', Iamblichus, 'The Life of Pythagoras', ch.12, in K.S.Guthrie, 'The Pythagorean Sourcebook and Library', p.30, Phanes Press, 1987

that of the Egyptian priesthood but without the recognition of any divine authority. The word for a mathematical demonstration in Greek, is *'deiknumi'*. This means, 'I show'. This sense was maintained in the term, 'demonstration', which was used until the 19th and 20th centuries when it was gradually replaced by the word, 'proof'. This seemingly small change was deeply significant. The word 'demonstration' offers an explanation which the receiver may or may not accept. There is a parity of status between the one who explains and the other who may accept or who may question. The word 'proof' claims certainty; the subtext being that if you don't understand it, there's something wrong with you[5].

This change in terminology coincided with the rise of expert elitism generally. This was one result of the rapidly increasing amount of information gathered (scientific and other 'data' [6]) from the 18th century on, which, in turn, resulted in increasing fragmentation of subjects of knowledge[7]. There are now specialists in thousands of different domains and very few generalists. Specialists often disagree amongst themselves but they all claim authority over anyone outside that circle. They mostly ignore Einstein's dictum, "If you can't explain it to a six year old, you don't understand it yourself". It is, of course, understandable that one might not understand something one has learned or discovered: quantum physics is a prime example. What is important, is to be honest about this.

The 'proof' power game was not the Pythagorean way since their ethos was to be respectful of others. It is a mistake to call them elitist simply because they had protocols for admittance to the community and because they kept the knowledge they gained, solely within the community. The protocols were a way both for the community and for a candidate seeking admittance, to assess

[5]Within mathematical circles, a proof is still seen as a demonstration which can be queried, but for those outside the charmed circle, it is not.

[6]'Data' means 'given' in Latin. In fact, this information is not simply given. It is sought for. And what kind of information we look for, depends on our prior ideas of what is relevant to our enquiry. There is no theory-free data.

[7]T.S.Eliot, "Where is the wisdom we have lost in knowledge? Where is the knowledge we have lost in information?", 'The Rock', play performed in the Sadler's Wells Theatre, London, 1934.

whether she or he was serious in their intent. It was not possible to buy one's way into the community[8]. The reason why technical knowledge was not to be divulged, was because Pythagoras knew that it was powerful and only those who had done the necessary, personal, spiritual work that was the basis of the Pythagorean community, could be trusted to use it responsibly. But amongst peers who had done this work, there was a communal sense of seeking to explain truths one had reached and to offer them for questioning and deepening.

We see that relationship is a key concept for the Pythagoreans. *Philia*, the relationship of loving friendship was the *raison d'etre* of their communal living and the source of their mathematical understandings, part of which was the fundamental relationship between two numbers, which we call, 'ratio' from Latin; in Greek it is *'logos'*. *'Logos'* also means 'reason' and logical reasoning is the basic relationship between consecutive statements in mathematical demonstration. We see that the way the Pythagoreans lived, how they observed nature and how they reasoned, formed a seamless whole predicated on love as manifest in harmonious relationships.

The word 'relationship' for the abstract concept we now have, did not exist in ancient Greek times. It was clearly operative in Pythagorean philosophy. We can now articulate it as a fundamental concept in 21st century Pythagorean mathematics. In fact *'logos'*, as the ratio between two numbers, could be seen to be the core of *arithmetike*, the Pythagorean study of the qualities of numbers, in that all numbers are ratios of one[9], as John O'Connor[10] observed in the BBC Radio 4 'In Our Time' programme on Pythagoras.

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[8]Iamblichus tells of a tyrant who tried to do this and was refused. 'The Pythagorean Sourcebook and Library', K.S.Guthrie, Phanes Pr. USA, 1987. In 'The Life of Pythagoras' by Iamblichus of Chalcis, pp.116–7

[9]And, of course, the 'Logos' is the 'Word' in the St. John's Gospel, 'In the beginning was the Word, and the Word was with God, and the Word was God'. This is the Divine Oneness of the Trinity. More on this in Nicola Graves-Gregory [NGG] 'Mathematics and Morality, Refining Awareness, Deepening Love', to be published 2021.

[10]John O'Connor and Edmund Robertson created and maintain the mathematical history website, "MacTutor",
`https://mathshistory.st-andrews.ac.uk/`

Looking at the original meanings of some key Pythagorean words, *mathemata, philosophia, logos*, has been enlightening. Let's look at the meanings of Pythagorean names. The name, 'Pythagoras' is made up of two parts. *Pythia* is the Pythian or Apollonian Oracle. Pythagoras' mother was called Pythais. *'Agora'* means forum or meeting place. The Pythagorean school was Apollonian in nature and it's openly in his name: Pythagoras created a forum for Apollonian thinking. The word *'agora'*, 'meeting place', chimes with *'philia'*, 'friendship' in 'philosophy', and emphasises the communal spirit that Pythagoras sought to bring into the world.

By looking for the meaning within names, we are following Iamblichus[11], one of the main chroniclers of Pythagoras' life. For example, he analysed the name 'Apollo' of the Greek sun god, into the constituent parts: *'a'*, meaning 'not', and *'pollo'* meaning 'many', which gives the meaning, 'not many', or 'the one'. The Apollonian religion was a shift away from the prolific, Greek pantheon of many gods and goddesses, to the belief in a fundamental, Divine Oneness. Pythagoras' wife's name was Theano. She was involved in the school along with other women. Her name means 'of the Divine'. The central aim of Pythagorean mathematics is communion with the Divine, Apollonian Oneness.

The *mathematikoi* or *esoteriki* were the inner, active members of the school. The reason why 'esoteric' in English carries the meaning, 'spiritual' or 'mystical', is because that was the nature of the inner, Pythagorean group. The word, *'esoterikos'* in Greek, meant then, as it still does, simply 'inner' [12]. Another group of the school were the *akousmatiki.* This means 'listeners': they were part of the school but, not having made the full commitment to live the principles, were not allowed to participate fully[13]. Note that they were called listeners and not observers. The hearing was more important than the seeing.

The core of Pythagorean teaching is number. But their con-

[11]Peter Kingsley, in 'Ancient Philosophy, Mystery and Magic', publ. OUP 1995, comments on the ancient Greek love of word play.

[12]In Greek now it carries both meanings.

[13]'The Pythagorean Sourcebook and Library, K.S.Guthrie, Phanes Pr. USA, 1987. In 'The Life of Pythagoras' by Iamblichus of Chalcis

cept of number was profoundly different from our modern concept. The change in the concept of number began gradually from the 3rd century AD onwards and underwent rapid acceleration in the 13th to 16th centuries[14]. It was this that made the subsequent, technological and economic revolutions possible, definitively marking the end of the Middle Ages.

To enter the world of Pythagorean number, requires a difficult act of imagination. This is not the same as fantasising. I'm fantasising if I daydream that I've won the lottery (I don't even buy a ticket). Such fantasising is very different from imagining in my mind's eye, a circle or sphere. True imagining is the primary, mathematical activity. The subject matter of mathematics is all imaginary, true imaginary! Plato used the term, 'ideals'. Geometric concepts are ideals. A point is a position with zero size. A line has no width. A plane has no depth. We are surrounded by lines and planes in our buildings and furniture but these are approximations to the ideals which only exist in our imagining minds. Similarly, with numbers: we have numerous [sic] examples of numbers in the material world; humanity these days does a lot of counting and measuring. But numbers themselves are not material.

Mathematics can describe elements of the visible, tangible world but it does so in terms of intangible and mostly invisible entities. Because mathematics has largely become identified with the technology it has engendered, most people, whilst acknowledging its abstract nature, are not actually aware of the nature of this abstraction. The mathematical world is one of imaginary entities, abstract concepts, and their interrelationships. Even most mathematicians do not think of the implications of the imaginary nature of mathematical worlds. The shift into that language happens very early in our lives so that it can seem as natural to us as our mother tongues.

This is, of course, true of our science also. From Newtonian forces to quarks and black holes, these are imaginary entities. They have a reality, in so far as scientists' calculations, on the basis of

[14]NGG, 'Historical Changes in the Concept of Number', ArXiv and ANPA Proceedings, No.34, 2014

assuming their reality, i.e. using the mathematics of the Standard Model of physics, achieve predicted results in the material world. But there are other non-mainstream theories that also fit these results[15]. All the theories are only possible, explanatory models. The reality of the imaginary, mathematical worlds is different from that of science theoretic entities: it depends not on results in the material world but on the internal, logical connectivity between these entities.

But notice that much of our everyday living is also in imaginary worlds. This is apart from the fantasising that we do. When we walk down streets that we know well, we mostly don't really see them. We may be on our mobile phone, or in some internal monologue, and we are only marginally aware of our surroundings, the buildings, the road, trees, people. We are not actually seeing what is around us. We are registering concepts, 'building', 'tree', 'person' etc sufficiently to keep our bodies safe, not to bump into things[16] but we are not actually seeing them, We are walking through barcodes, locked inside the open prison of our mother tongue. Hence the difficulty of thinking outside the box.

So, in beginning to investigate Pythagorean number, we find we need to investigate the nature of mathematical reality per se, which entails asking what we mean by reality per se. We are being drawn deep, but that is not really surprising. Pythagoreanism is holistic and holism is deep! "Whats it all about?", a dear friend used to ask me. And my answer was always immediate and always the same, "Love". I didn't know then that I was a Pythagorean and that this is the basis of Pythagoreanism.

I shall not attempt to define reality. Definitions are appropriate within symbolic mathematics but not in verbal languages.

[15]For example, Nick Thomas' counterspace model and Peter Rowlands' nilpotent rewrite system. These both preserve the standard nomenclature of the invisible, subatomic entities, but in their different theories these entities come into being in different, mathematical ways and, as such, have different ontologies. Life is overdetermined, as Althusser commented. Different stories or explanations are not necessarily contradictory, just different perspectives.

[16]Note that, regardless of the nature of being that we might be steering clear of, to the extent that our focus is internal, the beings are equivalent to inanimate things to us.

Mathematics is the most rigorous language that we have and basically still follows the template articulated by Euclid. As well as definitions, *horoi*, there are postulates, *aitemata*, and common notions, *koina*, which we take as understood. Different definitions, postulates and common notions result in different rules, structures and processes. The syntax dictates the strict logical connections between consecutive statements in mathematical demonstrations, and these are subject to peer review. This system ensures optimal consistency within symbolic mathematics. When inconsistencies or paradoxes arise, they may point ways to deeper understandings, as Imre Lakatos illustrates in his seminal work, 'Proofs and Refutations' [17].

In our living, verbal languages, strict definitions and purely logical reasoning is not appropriate[18]. We use words, and words come to us, in multiple ways and when we turn our attention towards the great wholeness of which we are part, we realise that it is beyond human comprehension, beyond words. This is the Pythagorean Oneness which contains all: all entities, all relationships, all processes. This great Oneness may be called by many names, the Monas, God, the Divine, Paramatman, Allah, the Universe, or others. This is the ultimate (and primordial) reality, which cannot be defined, only indicated[19]. In referring to this ineffable reality, from here on, I shall capitalise it, 'Reality'.

As regards the other, partial realities which we have briefly looked at, mathematical, scientific, consensus normal and individual, I shall describe them now as different realms of awareness, where awareness is our intelligent sentience beyond words. We observe the wide, intelligent sentience of babies and small children, which gradually gets focussed into narrower realms. Within these

[17] Imre Lakatos, 'Proofs and Refutations', Cambridge University Press, 1976

[18] Mathematical thinking, as opposed to formal, written mathematics, is also outside of these constraints. It is a more open space, allowing new ideas to enter, which can then be tested more rigorously. This is the key theme of Jacques Hadamard's delightful book, 'The Psychology of Invention in the Mathematical Field', publ. Princeton Univ. Pr., 1945.

[19] Alfred North Whitehead explicitly chose to describe, rather than define, verbal language terms in the process philosophy he developed when he was at Harvard.

realms we discover or create concepts, thus creating worlds of consciousness within the realms of awareness[20]. Ineffable Reality contains all these realms and more. Beginning from this Ineffable Reality, I posit an ordering of human cognition, a 21st century Pythagorean epistemology outlined below. Read '>' as, 'is greater than and includes'.

Reality > our awareness,
Our awareness > our consciousness,
Our consciousness > our language,
Our language > our reason,
Our reason > our logic

Mathematics is a language.

George Spencer Brown observed that "The more a being cultivates consciousness at the expense of awareness, the stupider it becomes" [21]. Awareness is our human access to Reality. Awareness can function within different realms, dependent on temporary circumstances and the more general circumstances of our growth and interest.

It is important to remember that, as Lakoff and Nunez point out, "All knowledge is human knowledge" [22]. Our shared knowledge is consolidated in language. Our individual knowings come from our personal, individual, perceptual awareness which is sensory and extrasensory (and we have many more than 5 physical senses, as materialist science now confirms), also from our social interactions with other human becomings[23], in the specific world into which we are born. In order to develop as individuals, and thus for humanity to develop, we need to find and create ways to open our awareness

[20] This is a recurrent theme in Rudolf Steiner's 'Die Philosophie der Freiheit', 1894, transl. 1964, Michael Wilson, 'The Philosophy of Freedom'.

[21] G.Spencer-Brown, 'Laws of Form', 1st publ. 1969, 1994 Edn, Cognizer Co., footnote p.ix. Also, "Western civilisation has promoted consciousness and neglected awareness almost to the point of complete idiocy."

[22] Lakoff and Nunez, 'Where Mathematics Comes From', publ. Basic Books, 2000.

[23] I use the term 'human becoming' to point to the inherent desire within human beings to develop, to evolve, to become more human. That this is possible for us within our incarnate forms, appears to be a key factor differentiating us from other living beings with whom we share this world.

to more inclusive realities, approaching Reality[24].

I have been calling these partial realities 'imaginary' to emphasise that they are non-material, involving our imagination, more than our physical senses. I could use other words, 'conceptual', 'mental', 'intellectual', 'spiritual', 'fictional', 'theoretical', 'notional', 'metaphorical', 'imaginal'. Clearly these all have different connotations. For the present I shall stay with 'imaginary' and I hope I have described my meaning sufficiently. Now I want to examine our ideas of 'knowing'.

In normal, consensus, material reality we have two main types of knowing. In many non-English languages there are two different verbs for these. In Romance languages such as French, these are 'savoir' and 'connaitre', in Italian, 'sapere' and 'cognoscere', in Spanish, 'saber' and 'conocer'. This Is the case also in German, 'wissen' and 'kennen'. The former of each corresponds to the scientific world (Latin, 'scire', 'to know'), which is knowing about things; the latter to the world of relationships, relating to other beings and ways of being with things.

Our modern, electrotechnical world has developed by elevating knowing about things, above relational knowing. This is part of what Spencer-Brown describes as elevating consciousness above awareness[25]. Martin Buber, in his book, 'I and Thou', describes at length these two different modes of being in the world[26]. The world of knowing about things, is the 3rd person world: we are the active subjects, the Newtonian style observers, who survey the activities of things, and work out how to make things happen whilst being seemingly unaffected ourselves. Buber names this, 'the world of *It*', which is dead. He shows how, in taking this stance, seeing the world as composed of things, of objects, we are actually affected: we objectify ourselves; we kill the life in ourselves. He contrasts

[24]Mingyur Rinpoche speaks of this greater awareness as open awareness, that we can approach through meditation.

[25]G.Spencer-Brown, 'Laws of Form', 1st publ. 1969, 1994 Ed'n, Cognizer Co., "Western civilisation has promoted consciousness and neglected awareness almost to the point of complete idiocy." footnote p.ix.

[26]Martin Buber, 'I and Thou', authorised transl'n, Ronald Gregor Smith 1937, from 'Ich und Du', Berlin 1923, 2004 ed'n publ. Continuum, London, UK

this with 'the world of *Thou*'. This is the Divine *Thou*, the Divine without who can waken the Divine reality within us (where we relate to any and all other as *Thou*[27]) where we and all, are alive[28].

Consciousness kills; in awareness we are reborn[29]. Clearly we do need consciousness in order to make maps of material and mental terrain, and to communicate with other human becomings. When we venture into the unknowns that awareness makes available, we need to discover/create concepts, map elements, Ariadne's threads, to help us orient ourselves, to find/create paths for ourselves and for other human becomings.

But we have now constructed such a vast, seemingly all-encompassing world of *it*, that we have lost sight of what it was all for, in the first place. Materialist science, the pursuit of the knowledge of *It*, has the repeatable experiment at the heart of its methodology. Repeatable is mechanical. Life is not. The elevation of consciousness over awareness, has resulted in the elevation of experiment over experience.

This cancer-like growth of the world of *It*, in Western, cultural mentality, has been matched in physical reality by an exponential growth of movement of people from the land to cities, together with the expansion of cities from millions to over 100 million inhabitants in places[30]. And this has been matched by an exponential growth of mass media. As human lives have been increasingly boxed into cities, and cut up into clock-time, work/leisure compartments, so this leisure has been increasingly dominated by passive consump-

[27]This is a recognition of the Divine within the other, as in the Hindu and Buddhist greeting, *'Namaste'*, meaning approximately "I salute the Divine in you".

[28]William Blake, "Everything that lives is holy". Pythagoras, Buber and NGG add, "Everything lives".

[29]Again this is a theme that runs through Rudolf Steiner's 'Die Philosophie der Freiheit', 1894, transl. 1964, Michael Wilson, 'The Philosophy of Freedom'. There is also a strong resonance here with, "For as in Adam all die, so in Christ, shall all be made alive.", 1 Corinthians 15.22, The Bible, King James' Version.

[30]The cancer metaphor is clearly justified with respect to cities as demonstrated by Geoffrey West in 'Scale, The Universal Laws of Life and Death in Organisms, Cities and Companies', publ. Weidenfeld and Nicholson, 2017. On pp.290 ff. his quantitative analysis shows the fractal nature of city development and the similarity with bacteria.

tion of mass media.

This is combined with the increasing pervasiveness of the belief in materialism. As we know, materialist science has been phenomenally successful in the way intended, in the world of *it*. The technological success of materialist science resulted in overspill. The belief in materialist science is now the default, total theory of reality. Its creation story begins with 'the Big Bang' which is beyond materialist scientific explanation to explain so it is just stated as 'fact'. In effect the story is, "Give us one miracle and we can explain everything afterwards in materialist scientific terms". If this were stated explicitly, that would be honest. It would be clear that one could choose to believe this, or adopt other beliefs. Unfortunately it is not openly stated. It is obscured, and as such, it is an ideology, similar to the histories of nation states, each of which is taught as 'fact' when all are mutually contradictory[31].

It is an ideology that has fostered the hubristic belief that humanity is the pinnacle of intelligence in the universe. Unfortunately Pythagoras' school was broken because his message of love was seen as politically dangerous[32]. This meant the loss of the meta-idea of his teaching: that for all knowledge of *It* that we acquire, we need to develop personally, spiritually, in order to use the knowledge responsibly. As a result we have been living in the world of the Sorcerer's apprentice whose creations have been at least as destructive as they have been helpful, possibly even more so overall[33].

Materialist science also has explanatory problems at the other end of its cosmological story, with human consciousness. The claim has been for over a century that materialist science will explain

[31]One example: in Great Britain Sir Walter Raleigh is known as a great adventurer. In Spanish history he is considered to be a pirate.

[32]Iamblichus tells of the persecution of the Pythagoreans in 'The Pythagorean Sourcebook and Library', K.S.Guthrie, Phanes Pr. USA, 1987. In 'The Life of Pythagoras' by Iamblichus of Chalcis, pp.116–7. Also Peter Kingsley, 'Ancient Philosophy, Mystery and Magic', publ. OUP, 1999, p.322, writes "With the dispersion of the Pythagoreans which evidently happened in the mid-fifth century as a result of the dramatic attacks and oppression they suffered in Southern Italy, ..."

[33]The extent of environmental damage caused by human technological advances is now dangerously clear, as is the result of technological 'advances' in weaponry.

how consciousness arises from matter. The neuroscientific research to this end, is similar to the 18th century attempt to locate the human soul in the body by cutting it up. This failed for the same reason: matter is of a different nature from soul, consciousness and awareness. The latter concern our subjective experience of being in the world, not of ourselves as subjects of experimentation. When we truly meet other beings in the world, we are in the world of *Thou*. Our human 'I' meeting another 'I' is the phenomenological moment of reality[34]. These moments are life-changing, sometimes in very small, or seemingly only small, ways.

From the perspective of materialist ideology, the world of *Thou*, the world of subjective experience is secondary. This is the logical conclusion of Galileo's calling quantitative measure primary: he believed that he was eradicating any taint of the physical senses, but in fact he was simply elevating the visual sense above the others[35]. Materialism has been the dominant ideology in Western schools for over a century. For people who have not had the possibility of an alternative view in their home life, for example, from a religious family, they believe the materialist story without realising this is a belief. For those who can make a success of their lives within our materialistic culture[36], this does not present immediate problems (although it may later in life when outer success no longer satisfies). For those who struggle in our money dominated world, they can easily see themselves as failures, and life seems meaningless. They believe that human consciousness is merely an epiphenomenon of random events from a 'Big Bang' billions of years ago. This is the psychosocial damage of materialism, which manifests in many

[34] As Jon Maynard Gregory puts it: "When the Cosmic Impulse intersects with the Great Ocean of Being" (personal conversation).

[35] Jeremy Naydler, 'The Future of the Ancient World, Essays on the History of Consciousness', publ. Inner Traditions USA, 2009, p.16 and the whole of the essay, 'The Restitution of the Ear', pp.6–20.

[36] US Senator Robert Kennedy: "The Gross National Product does not allow for the health of our children, the quality of their education or the joy of their play. It does not include the beauty of our poetry or the strength of our marriages, the intelligence of our public debate or the integrity of our public officials. It measures neither our wit nor our courage, neither our wisdom nor our learning, neither our compassion, nor our devotion to our country. It measures everything, in short, except that which makes life worthwhile."

ways, in depression, other mental health problems and even child suicides[37].

Within Pythagoreanism all our lives are meaningful as part of the transcendent and immanent, Divine Oneness of life and the universe, whose origin is a great mystery. We acknowledge ourselves as living beings-becomings, with other living, beings-becomings, all parts of a living universe. In this awareness, we attempt to relate to others as *Thou*, in Buber's sense. The uniqueness of Pythagoreanism is that it also researches the world of *It* through number. Most religions are solely moral and ethical. The only other tradition approaching this dual, spiritual and scientific aspect is Hindu Vedanta[38].

To Summarise: the importance of attempting to revive Pythagorean mathematics now is precisely because of its holistic philosophy.

In the 21st century we are in the obverse situation to that of two and a half millennia ago. That community was deeply embedded in the world of *Thou*. Somehow, astonishingly[39] they created a way to embrace the world of *It* also, through number. Now in the Western, materialist dominated world we are deeply embedded in the world of *It*. Might Pythagorean number help us open a door the other way? Might it help us bridge the worlds of *It* and *Thou*?

The word for 'number' in ancient Greece, was *'arithmos'*. These numbers are not the ones we are used to now. Zero did not exist. There were no negative numbers, no decimals. The early Greek

[37]Viktor Frankl's experience in a concentration camp in Germany showed him the importance of believing our lives are meaningful. He wrote about this in 'Man's Search for Meaning', publ. Rider, 2004; original German publ'n 1946.

[38]There are three main streams of Vedanta. Advaita (non-duality) states that ultimate reality is oneness; Dvaita that it is duality; Vishishtadvaita states that ultimate reality is oneness but consensus, material reality is the world of duality. Vedanta includes the world of 'It', but researches health (Ayurveda) more than science.

[39]We know Pythagoras travelled. Peter Kingsley's 'A story waiting to pierce you', publ. The Golden Sufi Center US, 20010, widely and deeply referenced, tells a very different story of the nature of Pythagoras' travelling from the usual, academic one.

numbers are whole numbers, the natural numbers[40], the positive integers. The word 'integer' relates to 'integrity', coming from the Latin, *'integritas'*, meaning 'soundness, wholeness, completeness', from *'in'*, 'not' and *'tangere'*, 'to touch' [41]. We have the phrase "sound as a bell". A bell may look beautiful but if it has an invisible crack it will not ring clear. The *akousmaktiki*, the 'listeners' on the edge of the Pythagorean community, had to listen for the wholeness, not be dazzled by surface images.

Also one is not a number! It is Oneness, the great Oneness that contains all. In this sense, One is the biggest number. The Pythagoreans called it the *Monas*, the Divine Oneness. Although this sounds strange to us, it is reasonable; multiplicity only begins with two. Two is the first number. As I said, it is difficult for us to imagine this reality.

There were two aspects to the Pythagorean study of numbers, *arithmetike*, which concerns the qualities of numbers, their *eide*, the 'forms', and *logistike*, which concerns the quantitative aspect of number, the *hule*, 'amount', and the practical skills of calculating. Clearly there is a synergy between *arithmetike* and *logistike*. The concepts 'odd' and 'even' come from multiplication. The concepts, 'triangular', 'square' et cetera give visual meaning to numbers which result from different calculating procedures. This was a living synergy for the Pythagoreans. In the following centuries quantitative mathematics, *logistike*, saw almost continuous expansion with extraordinary advances in the 16th and 17th century AD which were only possible because of the deep changes in the concept of number[42] that had occurred.

In contrast *arithmetike*, was left behind. Its technical mathematical side became our present day number theory but the fundamental concepts of number theory did not expand to embrace the

[40]Leopold Kronecker famously said, "God created the integers; all else is the work of man."

[41]Latin, *'integritas'*,figuratively, 'purity, correctness, blamelessness'. This gave rise to 'integer' as 'whole number' i n the 1570s `www.etymonline.com`. Interestingly, this was at the time when, within mathematics the consciousness of the importance was lost. It was preserved within this word.

[42]See NGG, 'Historical Changes in the concepts of number, mathematics and number theory', ANPA Proceedings, 2014 and ArXiv:1705.02386.

extraordinarily complex, structural developments that occurred in *logistike*[43]. Up until the 20th century number theory was seen as a harmless pastime of mathematicians. It was only considered more generally important when cryptography made it useful in the business world and Riemann's theory showed the astounding, unexplained connection between primes and quantum theory.

The original, Pythagorean qualities were not just technical. They included also qualities such as, justice, male, female etc. This metaphysical side of *arithmetike* was excluded from serious, mathematical investigation, named 'numerology' and dismissed by academia. Certainly these associations do not fit our rationalist idea of mathematics but perhaps we need to rethink our perspective, remembering that Pythagorean mathematics comes from the wisdom of love, the world of *Thou*. Our normal usage of quantitative number is clearly in the world of *It*. Might we find a bridge with the world of *Thou* in a wider idea of qualitative number?

Numbers are generally thought of, as being abstracted from instances, counting sheep etc. But what might it mean to think of our experiences of numbers as instances of their reality? For example, if we think of the number four, we can think within *logistike* of how four is made up mathematically, additively as $1+3$, or $3+1$, or $2+2$, multiplicatively as 2×2, or 2 squared. We may also think of the four seasons or the four directions or a musical quartet or 4:4 time in music. Could we begin to feel the number four similarly to the way that Ramanujan felt so many whole numbers by appreciating their purely mathematical characteristics? What kind of feeling is this? Might this lead to metaphysical concepts being associated with numbers? Anthony Blake gives many such examples in 'Meaning of Numbers' [44], including C.S.Peirce's writing of numbers' qualitative significance as categories of understanding[45].

Perhaps we can ease open some spaces in, between, or beyond, our present ideas of qualities of number. Given that both our math-

[43] For example, Euler characteristics.

[44] Anthony Blake, 'Meaning of Numbers', pp.69–123, in 'Contexts', Proceedings of ANPA 31, Ed. Arleta D. Ford, 2011

[45] *Ibid.* pp.95–6, C.S.Peirce, 'A Guess at the Riddle', 'Essential Peirce Vol.1', Wiley 1992

ematical world and the quantum world are outside of consensus, material reality, outside the parameters of our normal understanding, I propose that we attempt to imagine a realm between *It* and *Thou*, a transit world which may be inhabited by some qualities of numbers yet to be discovered and/or created.

Earlier I adopted the term 'imaginary' to describe the mathematical world. I also listed other words which I might have chosen: 'conceptual', 'mental', 'intellectual', 'spiritual', 'fictional' [46], 'theoretical', 'notional', 'metaphorical', 'imaginal'. Interestingly the German word, *'geistig'* carries the meanings, 'intellectual' and 'spiritual'. Nearly all the other words belong to the world of *It*. 'Imaginal' is the exception.

Henri Corbin coined this word, in his paper '*Mundus Imaginalis* or the Imaginary and the Imaginal' [47]. He was studying Arabic and Persian texts of real, spiritual experiences and wanted to describe "a very precise order of reality", corresponding to "a precise mode of perception" and as such he did not want to use the term, 'imaginary'. In his paper he gives "a brief outline" of the "order of reality" of the *mundus imaginalis*, or 'the eighth clime', as the Islamic theosophers named it, and then studies "the organ which perceives it, i.e., imaginative consciousness, cognitive Imagination".

The Persian text he refers to, includes precise, mathematical descriptions of, for example, "the supersensible world of the Soul or Angel Souls" which starts "at the convex surface of the ninth sphere". There is a subsequent return "from the outside to the inside". It is remarkably similar to the Riemannian space which Dante enters in the 'Divine Comedy' [48]. Both narratives are also basically dialogues between the visionary and supernatural beings.

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[46] Hans Vaihinger wrote of the importance of fictions in human life in 'The Philosophy of 'As If' ', transl. C K Ogden, publ. Harcourt, Brace and World, 1925. Note also the similar etymological roots of 'fact' and 'fiction'; the former from *facere*, 'to make, do...; the latter from *fingere*, 'to fashion, form, mould...', Cassell's Latin Dictionary 1946

[47] `http://www.bahaistudies.net/asma/mundus_imaginalis.pdf`. This is the 1972 version (condensed with the permission of the author) of Henri Corbin's paper, delivered at the Colloquium on Symbolism in Paris in June 1964.

[48] Robert Osserman described the Riemannian geometry of Dante's space in 'Poetry of the Universe', publ. Doubleday NY, 1995

In this sense they belong to the world of *Thou*, whilst also belonging to the world of *It* in their numbering of levels of reality in these Riemannian spaces.

Corbin speaks of imaginative consciousness or cognitive imagination, being possible ways to enter this mathematical, liminal realm between *It* and *Thou* ? Georg Kuhlewind proposes 'cognitive feeling' [49]. This is embedded in Rudolf Steiner's analysis of our human modalities within material reality as threefold: thinking, feeling and willing, where willing engenders physical interaction. The importance of this explicit recognition of our bodily being in the world is another way that Pythagorean mathematics of the 21st century AD differs from that of the 6th century BC. This is also different from our normal ideas of mathematics. It is elaborated in Lakoff and Nunezs book, 'Where Mathematics Comes From, How the Embodied Mind Brings Mathematics into Being' [50], a tour de force of cognitive science, showing how mathematical ideas are rooted in physical experience. A further level is given by Armin Husemann's 'The Harmony of the Human Body, Musical Principles in Human Physiology', a Pythagorean tour de force from an author who is a medical doctor and musician, following Steiner's analysis, showing how the form and development of the human body manifest mathematics and music[51].

We are being led to look at Pythagorean geometry, the mathematical study of space, our physical experience of space giving rise to real, imaginary space[52]. Geometry is distinct from *arithmetike* and *logistike*, not only because adding and subtracting, multiplying and dividing, are different from drawing, but for a deeper reason. Lines are qualitatively different from numbers. Numbers, *arithmoi*, are discrete: they cannot be divided, as they would then no longer be whole. But lines are continuous and infinitely divisi-

[49] Georg Kuhlewind, 'The Gentle Will, Guidelines for Creative Consciousness', transl. Michael Lipson, publ. Lindisfarne Books 2011

[50] Lakoff and Nunez, 'Where Mathematics Comes From', publ. Basic Books, 2000.

[51] Armin Husemann, 'The Harmony of the Human Body', transl. Christian von Armin and Alan Stott, publ. Floris Books, 1994. German ed'n publ. 1989.

[52] Note that even in Dante's and the Persian imaginal spaces, the visionary is in a body.

ble. The study of geometry was the study of space as consisting of points, straight lines, flat planes and solids. *Arithmetike* included the study of ratios and proportions (ratios of ratios) but not of fractions. These were only allowed in practical work. The polarity, discrete and continuous or whole and divisible, has deep, philosophical implications which are totally obscured in our present concept of linear, decimal number. David Bohm's concern for wholeness is in the title of his book on quantum physics, 'Wholeness and the Implicate Order' and the first chapter is entitled 'Fragmentation and Wholeness' [53]. A loaf of bread is not the same as a pile of crumbs.

The Pythagorean school was broken and the Pythagorean diaspora continued their work in different ways but it was only the technical aspects of their mathematics that spread into the wider world. The core of their holistic philosophy was gradually lost. In the 1st century BC Cicero managed to revive it to some extent by promoting the seven Liberal Arts as a foundational education for Roman citizens. These consisted of the trivium, the three verbal arts, grammar, logic, rhetoric, together with the quadrivium, the four Pythagorean, mathematical arts, number, geometry (number in space), music (number in time) and astronomy (number in space and time).

Rome fell and it was only centuries later in the school of Chartres that the Liberal Arts again flourished with Pythagorean mathematics at the centre. Meanwhile in neighbouring Italy, Fibonacci was introducing and popularising, the symbol which would marginalise the whole Pythagorean endeavour. This was the symbol, or sign, for zero[54], the foundation stone of the Hindu-Arabic numeral system whose introduction revolutionised calculation and changed the European world..

[53] David Bohm, 'Wholeness and the Implicate Order', publ. Routledge, Kegan and Paul Ltd., 1980, pp.1–19.

[54] Signs and symbols are different. Signs have a $1-1$ correspondence with the meaning they represent and operate at a conscious, intellectual level. A symbol has one form with many meanings and operates at a subconscious, psychological level. Many of our signs are also symbols, e.g. 0, 1, $+$, $\times$. The effects of symbolic meanings of signs in consensus reality, is one theme in NGG 'Mathematics and Morality, Refining Awareness, Deepening Love', publn., 2021.

Up until that time in Europe, people used the Roman numeral system. This does not have a zero, and as such, it is not a numeral place system. The decimal numeral system that we take for granted today, has 10 symbols, 0, 1,2,3,. . . ,8,9 and we differentiate further numbers by assigning meaning to the places where the numerals are written. We start with the unit's place, then the 10's place, then the 100's place, then the 1000's place, going on as far as anyone wishes. As children, lots of us have written numbers with lots and lots of noughts, just because it's so amazing that we can!

In the Roman system there were individual signs for some of the numbers, starting with I for one, V for five, X for ten, L for 50, C for a hundred, D for 500, M for 1,000. Note that they did not need more. The other numbers were constructed according to certain patterns of these signs. For example, three is written III, four is written IV, 12 is written XII, 24 is written XXIV, 87 is written LXXXVII, et cetera. Imagine how difficult it was to calculate in this number system. Try multiplying 7×16 as VII times XVI. Monks spent weeks calculating the date for Easter (a movable feast) each year.

It's the zero that makes the decimal place system possible. Initially it was simply a record of the empty place on the abacus at the end of a business calculation. Fibonacci's book, published in 1202, was called 'Liber Abaci' ('the book of the abacus'). This new numeral system entered Europe as a result of the trading that had developed with Arabic countries during the Crusades. Ironically these exhausted the resources of the feudal lords and the Church and simultaneously engendered the emergence of new economic, social, cultural and political forms.

The deep, ontological difference between the Pythagorean notion of number and that which the Hindu-Arabic numerals ushered in, is that the latter are infinitely divisible. The identification with line was irresistible and now we have all grown up with the idea that this is the natural way to represent number[55], with the be-

[55]Lakoff and Nunez, in 'Where Mathematics Comes From', Publ. Basic Books 2000, investigate extensively metaphors and their interrelationships in mathematics, in particular, the Number-Line blend and the Numbers are Points on a Line metaphor pp.70–71, 295–296, 298–299.

ginning of number being zero (not two, or even one, which some mathematicians after Pythagoras had begun to accept). We see graphs as a natural way to show the quantitative relationship between different things. But this was a 17th century invention of Descartes, using Hindu-Arabic numerals.

This conflation of number and line now seems like common sense to such an extent that we see space as having a 3-dimensional Cartesian reality. The introduction of perspective in Renaissance painting in the 15th century began this process. The vanishing point can, in fact, be seen as a representation of a potential zero, one which is never actually reached[56]. Our visual sense has incorporated this 3-D mathematical form. Or we could say that this form has invaded our normal, visual sense. 3-D is, in fact, not natural[57], any more than clock time is.

Many mathematicians were aware that there was no ontological foundation for the new concept of number[58] but the attraction of the new, mathematical possibilities overrode all the seeming quibbles. And in the practical world of business and industry, philosophical doubts had no place and were ignored. This was the beginning of pragmatism in mathematics and a totally new world view, which promised progress and brought fragmentation in its wake[59].

There's an interesting paradox in the relationship with past and future, between the scholastic beliefs and those of the Enlightenment. The scholastic period held a belief in an earlier golden age

[56]The relationship between potential zero and potential infinity and the psychocultural implications of an 'actual' zero and centuries later, Cantor's 'actual' infinities, are explored in NGG, 'Mathematics and Morality, Refining Awareness, Deepening Love', awaited publication 2021.

[57]Recognising our incarnate, human reality, 21st century Pythagorean mathematics investigates our 6 directions, the three pairs of polarities, up and down, forwards and backwards, left and right.

[58]In fact, the Pythagorean ontology had already been eroded since Diophantus' work in the 3rd century AD. Zero and the number line were just the last straw! See NGG, Historical Changes in the concepts of number, mathematics and number theory ANPA Proceedings, 2014 and ArXiv:1705.02386.

[59]As C.S.Lewis said, "We all want progress... If you are on the wrong road progress means doing an about-turn and walking back to the right road and in that case the man who turns back soonest is the most progressive man." 'The Case for Christianity', publ. Macmillan NY, 1944, originally a radio broadcast.

of understanding to which seekers of knowledge aspired, but they also embraced teleology, the idea that things can be pulled by the future, Aristotle's final causes. Enlightenment thinkers introduced the idea of progress, the belief that each generation understood more, knew better than the previous one, but their science was wholly based in mechanical causality, the belief that the present is totally pushed from the past, a form of Aristotle's efficient causes. We can easily understand this belief in the omnipotence of causality, given the technological power of the new mathematics which emerged in the 17th century.

It was around 1600 that the new, symbolic language of mathematics emerged as an independent entity from the womb of verbal languages[60]. It was a lengthy pregnancy, requiring a number of factors to make it possible. The first of these was the introduction of the sign for zero and the Hindu-Arabic, decimal, numeral system which facilitated calculation. This was needed for the conceptual leap whereby equations were no longer seen as formulae to uncover a determined unknown, x, rather that this x could be a variable and the equation could describe a relationship between a changing variable and a result[61]. This was the birth of modern algebra.

On the basis of this and the scaffolding of Cartesian geometry and new thinking about infinity and infinitesimals[62], Newton and Leibniz were able to conceive of the possibility of creating equations whereby rates of change could be calculated. The differential calculus was born, leading to three centuries of extraordinary, rapid, technological and financial developments, with all their concomitant, extraordinary, rapid changes in human societal relations and relationships with our environment, this beautiful planet.

[60]Martin Lowe, 'The Introduction and Development of Symbolic Representation in Mathematics during the Late Renaissance' in 'Chiasmus', ANPA Proceedings 37 & 38, Ed. John C. Amson, 2018. Also Jacob Klein, 'Greek Mathematical Thought and the Origin of Algebra', transl. Eva Braun. Original German, part I 1934, part II 1936, English ed'n 1968, Massachussetts Institute of Technology.

[61]*Ibid.*

[62]The strange nature of infinitesimals is important and, like the nature of zero, was mostly not acknowledged at the time. There were exceptions but they were ignored. For more details, see NGG, 'Historical Changes in the Concept of Number', ArXiv and ANPA Proceedings, No.34, 2014

These immense cultural changes were predicated on the Enlightenment change in the very idea of what it meant to understand reality. As we noted, Galileo heralded this change, and with it key factors of the Renaissance, when he described the book of nature as being written in mathematics, by which he meant quantitative mathematics based on visual measure. The visual sense only functions at a distance. The elevation of seeing, leads smoothly to the separation of seeing other as *It*[63], rather than addressing the other as *Thou*. For the scholastics of the Middle Ages, as for Pythagoras and Buber, the ultimate *Thou* is the Divine. In their desire to understand reality, they asked, "What are the different, essential natures of things or beings in the world?" The question behind this, was "How do these manifest the Divine?"

The equivalent question that began in the Renaissance and Enlightenment times, and continues today, is "How do things work?" The codicil to this question is "How can we make things work for us?" This was encapsulated in Francis Bacon's often quoted (somewhat inaccurately), "Knowledge is power". His intentions (like so many that pave the way to Hell) were good. He believed that the new, technological science ('scientia') would improve the lives of all people.

This belief has continued to wreak havoc for centuries since. It rests on the mistake that improvement in human life requires only material improvement. David Bohm said that this was what he initially thought science would achieve; he later realised this was not true[64]. The new, Enlightenment science and the materialist ideology it generated, lacked the acknowledgement of the great gift of life in this amazingly complex and beautiful world. This is gross ingratitude; and hubris replaced humility in too many hearts and minds[65].

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[63]In Jeremy Naydler, 'The Future of the Ancient World, Essays on the History of Consciousness', publ. Inner Traditions USA, 2009, 'The Restitution of the Ear', on p.15, there is a copy of a Durer woodcut illustrating this beautifully.

[64]David Bohm, in an interview in 'Infinite Potential: the Life and Ideas of David Bohm', `https://www.youtube.com/watch?v=XDpurdHKpb8`

[65]Shakespeare, "All's well that ends well", Laseu, "They say miracles are past; and we have our philosophical persons, to make modern and familiar,

After three centuries of mathematical, scientific and technological progress based on the differential calculus, the steam began to run out. By the end of the 19th century, the 'fin de siecle' [66], there was a general, cultural shift. Problems began to show up in the foundations of the purely abstract, technical, mathematics, also in the mechanistic science based on this almost totally quantitative mathematics. In the 20th century, new mathematical disciplines arose including more qualitative ones, non-Euclidean geometries, non-Aristotelian logics, topology, catastrophe theory, chaos theory, complexity theory.

In physics, relativity replaced Newtonian gravitational theory. And discovery of the quantum world might have ushered in an expanded, human recognition of the importance of the world of *Thou*, since the observer could no longer claim to be outside the experiment; observation itself was seen to be an intervention. The physicist was not just looking in at Nature, like a game of billiards, recording measurements and attempting to deduce patterns of correlations between events. He (usually he) was barging into Nature's house, bashing things around.

In pre-industrial farming, it was understood that when we work the land, we needed to respect the invisible beings that we would disturb by digging. Prayers might be said, or some kind of ceremonial act performed. Similarly we knock on someone's door in the hope they might answer, perhaps invite us in. Leading physicists, Pauli, Schrödinger, Heisenberg, Bohr and later David Bohm, amongst others, did pursue deeper questions in response to quantum discoveries, mostly turning to Eastern philosophy. But as is normal in human behaviour, institutional science (locked into the world of *It*) found ways to contain these radical developments within the old paradigms and refused to look at the far-reaching

things supernatural and causeless. Hence is it that we make trifles of terrors, ensconcing ourselves into seeming knowledge, when we should submit ourselves to an unknown fear."

[66] It is noteworthy that, in English, this period, at the end of the 19th century, going into the 20th, is given this special name, not the 18th to 19th or other, previous ends of centuries. The fin de sicle was one of profound change in all cultural areas, literary, in the visual arts, as well as in science and mathematics.

implications[67].

David Bohm points out that "nature will respond according to the theory with which it is approached" [68]. We need to develop theories which are respectful of this wondrously beautiful, intelligent universe in which we are privileged to incarnate, and of the wondrous beauty and intelligence of the other beings sharing this incarnate time with us. This is a core element of the Pythagorean telos.

We have looked briefly at some other elements of what a 21st century Pythagorean mathematics and philosophy would entail. Clearly a revival, or renaissance, of Pythagorean mathematics is a radical project. The word, 'radix' is Latin for 'root'. Pythagorean mathematics is not an add-on app. We begin with ourselves as whole becomings in the whole universe of whole becomings, co-existing and interacting. Cognising our nature as becomings, not only beings, is a key difference between the original Pythagorean philosophy and mathematics and its possibilities now. Now we need to embrace change, process as part of Nature and part of our nature.

The Enlightenment differential calculus works brilliantly as a tool for describing quantitative, mechanical change. In the 20th century new mathematics emerged, seeking to embrace qualitative change[69]. To the extent that they remain firmly within the world of *It*, we, as participants in life are not engaged. We remain in the Newtonian world. For us now, in the 21st century AD, we vitally need to keep in mind the complementary aspect of the Pythagorean perspective, the world of *Thou*. With respect to any idea we need to ask ourselves, what might this mean in terms of our relationships with others? What are the implications in terms of our attitude, our motivation, our intentions, our deeds?

[67] Thomas S. Kuhn, in 'Scientific Revolutions', comments that new theories do not replace old theories on their own merits. In time the proponents of the old theories die and their theories with them.

[68] David Bohm, 'Wholeness and the Implicate Order', publ. Routledge, Kegan and Paul Ltd., 1980, p.6

[69] Robert Rosen in 'Life Itself', publ. Columbia Univ.Pr., 1991, argues cogently that we have completed enough quantitative analysis and we need to address qualities

As Pythagoreans we realise our responsibility for all our actions. Our attention is an action; our intention is an action. As incarnate beings, we interact physically with the other beings with whom we share this universe, human, animal, plant, mineral, seen and unseen. Different life forms have different modes of communication, of intelligence, of sentience, of time scale. We need to learn how to converse in different ways with different beings. This is the meaning of Pythagorean holistic philosophy, the wisdom of love.

In this short paper I have not been able to enter all of the implications of this philosophy for mathematics and science. In my paper in the ANPA Proceedings 32, 2012, 'Human Mathematics: New Beginnings from the Pythagoras Theano Impulse', I listed some areas for research and development. I include that incomplete list here as an Appendix. Also, in the present volume of ANPA Proceedings, there are many examples of Pythagorean mathematics, mathematics that comes from love and some of them are explicit about their wider philosophical remit, for example, Françoise Chatelin, Barbara Gabrys, Michael Manthey, Pat Toms.

This is a beginning. I'll leave you with words from William Blake's oneness through duality: "If the doors of perception were cleansed, man would see everything as it is: infinite. For man has closed himself up till he sees all things thro' chinks of his cavern" [70].

APPENDIX :

21st century Pythagorean mathematical areas for research and development[71]

1. Explicit emphasis on the mathematical experience which complements the emphasis on results. (Process as well as product)

[70] William Blake, 'The Marriage of Heaven and Hell', composed between 1790 and 1793, just after the French Revolution.

[71] NGG, 'Human Mathematics: New Beginnings from the Pythagoras Theano Impulse' in the ANPA Proceedings 32, 2012.

A. Self-awareness: the intention to develop self-awareness in the act of doing mathematics.

i. This includes witness awareness:

a. Awareness that we are part of the wholeness of life.

b. Awareness of our motivations, explicating our beliefs (from the past) and our directions and desires (towards the future)

ii. It also implies whole human thinking, not just abstract head thinking, but also heart thinking, moral, visceral, multisensory, intuitive, imaginative thinking.

B. Ongoing clarification of the telos of human mathematics, including refining our senses and revisiting Platonic values.

i. Beauty: mathematics evokes a sense of wonder.

ii. Truth: demonstration (*deiknymi*) clarifies and deepens our understanding; it helps us refine our sense of truth and reasoning.

iii. Good: we refine our sense of the harmonies and wholenesses in life and develop abilities to create harmony as appropriate in different situations and different ways.

2. Development of mathematics from other senses than vision, for example:

A. mathematics of gesture (kinaesthetic sense), projective geometry, eurythmy, Laban and Indian dance, embryology, social interactions etc.

B. mathematics of hearing: music, rhythm, harmony, Vedic sounding etc.

C. mathematics of the interrelations of heterogeneous senses in the oneness of the self.

3. Development of mathematics from different areas of the external world, for example:

A. Ethnomathematics, the arts and crafts, knitting, weaving, pottery, architecture etc.

B. Studies of the natural world, biology, geology etc.

4. Investigating symbolic mathematics for human meaning, sacred geometry and music, Vedic mathematics, meaning of numbers etc.

5. Investigating mathematical language, for example:

A. Grammar: 3rd and 2nd person (for example Pogačnik cosmograms), prepositions, verbs, such as $\geq$, $=$, $\rightarrow$, functions etc.

B. Signs and symbols, logic and archetypes.

. . . . . .

These are not all separate and the list is not complete.
A journey of a thousand miles begins with a single step.
Clearly many people (only a few of whom I have mentioned)
have already taken many steps on the Pythagorean path.