The Beauty of Mathematics and the Mathematics of Beauty

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Foreword

This is a seed paper. It is part of a larger project, rediscovering and regenerating Pythagorean mathematics for the 21st century AD.

The dominant, mathematical mode since the Enlightenment elevated quantitative mathematics and mostly disregarded its complement, the mathematics of qualities. The holistic Pythagorean approach included both. The limitations of quantification became increasingly apparent in the 20th century.

When we begin to investigate Pythagorean mathematics afresh, we discover that in its complex profundity it contains an inherent impetus towards a deepening of individual life and harmonious social relations, both so vitally needed today. This is an impetus towards true beauty.

The beauty of mathematics is generally appreciated: the beauty of geometric form, in crystals, flowers, snowflakes, Fibonacci spirals of shells, the golden mean in architecture, Bach fugues: mathematics is everywhere. Mathematicians also appreciate the beauty of mathematical ideas per se, as in beautiful equations and elegant proofs.

But some kinds of beauty can not be described by a mathematics of numbers and ratios. Sunsets, mountains, nature generally; and human qualities such as generosity, fortitude, kindness, can all be described as beautiful. Might there be a different kind of mathematics which relates to the beauty which encompasses these? Surely Dirac would think so, given that he wrote, "Mathematics is the tool suited for dealing with abstract concepts of any kind and

there is no limit to its power in this field"[1].

Arguments about beauty have traditionally revolved around the question: to what extent is it inherent in the object observed, to what extent is it 'in the eye of the beholder' (or the ear or other sense organ)? Professor Semir Zeki (University College London) and his team investigated this from a neuroscientific perspective[2].Different groups of people were given different images and sounds to experience, while activity in different parts of their brains was monitored. They were asked to assess the beauty or ugliness of the stimuli. Their assessments were subsequently examined in relation to the neural activity in the different brain parts.

Not surprisingly there were cultural differences in the assessments of beauty, although there was some cross-cultural agreement with respect to the beauty of the golden mean in the human face. Nevertheless, what did stand out clearly was the relationship between the participants' experience of beauty or ugliness, and activity in specific brain areas. Appreciation of beauty correlated with medial and orbital, frontal cortex activity. The experience of ugliness correlated with activity of the amygdala and motor cortex.

The frontal cortex is the seat of higher brain functioning whereas the amygdala and motor cortex process more primitive reactions and are connected with aggressive behaviour. So experiencing something as ugly activates our primitive brain, whereas appreciation of beauty is a more sophisticated response. This is borne out by the history of both visual art and music, where successive generations have come to esteem things that were totally rejected when they first appeared. Mathematicians ecstatic response to the beauty of Euler's identity is incomprehensible to those who only see a string of strange symbols. Indeed, creation of beauty is one index of civilisation, as we generally understand it, and this requires perception of beauty.

E.H.Gombrich in his classic book 'The Story of Art' clearly advocates looking for beauty rather than looking for ugliness. He

[1]Preface to P.A.M.Dirac, "The Principles of Quantum Mechanics", 4th ed'n., publ. Oxford, Clarendon Press, 1967, p.viii.

[2]The neurobiology of beauty, Prof.Semir Zeki lecture, 17th January 2017, `https://www.youtube.com/watch?v=9YUq-y2yA6E`

writes refreshingly that he does not "think that there are any wrong reasons for liking a statue or picture" whereas he does think that "there *are* wrong reasons for disliking a work of art"[3]. This is also the moral of the story of the guru (some say Jesus, others, the Buddha) walking with his students. When they come across a dead dog, with maggots crawling all over its flesh, one of the students points to it as an example of putrefaction, but the guru says, "What beautiful, white teeth!".

Prof. Zeki's work does point towards a both/and answer to the initial either/or, perceiver/object question. But it also takes us further: it provides external, scientific evidence that the perception of beauty is a more advanced, human response to the world than the judgment of ugliness. So, deepening and extending our faculty for perceiving beauty, could be seen as part of evolving our humanity.

Could we train ourselves to develop this faculty? In our modern, Western culture this usually happens only in specific areas of individual interest, such as music or art or poetry – or mathematics! But for Plato, beauty per se, together with truth and the good, formed the trinity of ideals to which humanity should aspire. Prof. Zeki's research confirms Plato's appraisal. In the Hindu tradition also, developing our appreciation of beauty is seen as an integral part of human, spiritual development. In the Vedas, their ancient texts of wisdom, there is a mantra, *"Bhadram karnebhih srunuyama devan"*, 'may my ears perceive the divine'[4].

Note that this focussing on beauty is not a comparative. When we smell the heavenly perfume of a rose, we are opening ourselves to the experience of that moment. We are not comparing it with unpleasant smells. And Prof. Zeki's research shows that to engage in that sort of comparison, to look for ugliness, is a mistake as it takes us into more primitive, aggressive parts of ourselves. We recognise this in extreme examples. Racism, sexism, any form of elitism, often leads to behaviour that we rightly consider to be inhuman or subhuman. But here we see that the seeds of this

[3]E.H.Gombrich, 'The Story of Art', p.5, 15th edition 1989, 1st ed. 1950, Publ. Phaidon Press Ltd, Oxford

[4]This mantra occurs in the Mundaka, Mandukya and Prashna Upanishads.

already occur in seemingly harmless value judgments. This is a salutary lesson.

Comparison really only belongs with the quantifiable, those things that we can measure with a ruler, or as if with a ruler, a ruler that can be divided into smaller and smaller units. These are attributes such as taller or shorter, bigger or smaller, faster or slower etc. Clearly the beautiful/ ugly antinomy is of a different nature. Given that most of our thinking is structured by our languaging[5], we see here that it is unhelpful to speak of all antinomies as if they are on some kind of sliding scale or number line. And it is wise to concentrate on developing our appreciation of beauty.

In Pythagorean and ancient Greek mathematics generally, quantitative, continuous (divisible) measure, *'megethos'*, only applied to sizes of things, lengths, areas, weights etc. Number was a separate, mathematical study. Numbers, *'arithmoi'*, were the whole numbers, the positive integers, beginning with the number 2. Their study, *'arithmetike'*, was separate from that of geometry with its *megethoi*. *Arithmetike* was concerned with the qualities of numbers, odd, even, triangular, square etc and the ratios and relationships between them[6]. It also included ascribing number correspondences to concepts such as justice or marriage. These two strands were later separated into number theory and numerology, to the detriment of both. Note that one is not a number because it is not plurality. For the Pythagoreans, the *'Monas'* the Oneness, was the Divine, which was the telos of their mathematics.

After the fall[7] of the Pythagorean school, the importance of the whole/divisible, discrete/continuous distinction was increasingly obscured. The first, definitive step in this direction was in

[5]The term 'languaging' was coined by Werner Erhard, founder of the Landmark Forum personal development training, drawing attention to the way we get trapped in language. Thinking 'out of the box' of existing language, is rare and sought after, quintessentially in mathematics which is the art of original box making (discovering) par excellence.

[6]Plato famously said, "The mathematician does not divide". That is, mathematicians study ratios but not fractions, which are part of *'megethos'*, and only used in everyday life.

[7]Iamblichus tells of the persecution of the Pythagoreans in 'The Pythagorean Sourcebook and Library', K.S.Guthrie, Phanes Pr. USA, 1987. In 'The Life of Pythagoras' by Iamblichus of Chalcis, pp.116–7.

the 3rd century AD when Diophantus included fractions as number solutions in the equations he was working to solve[8][9]. Then, through the Middle Ages and the Renaissance, irrationals, negatives and imaginaries began to be used as numbers[10]. Around 1200 Fibonacci introduced a symbol for zero into European mathematics. This came as a result of the trade with the Arabic world which grew (ironically) during the Crusades. It was originally only a marker for an empty place when abacus results were recorded. The cumulative effect of all these gradual extensions of the concept of number, paved the way for the final obfuscation, the conflation of number with line, the creation of the number line, which became the norm to the extent that it is now generally and misleadingly, called the 'real line'[11].

These changes had profound, philosophical implications which were discussed by some mathematicians at the time but were basically put aside, because of the immense possibilities of mathematical development they offered. This was all part of the growing, pragmatic paradigm which culminated in the humanist[12] philosophy of the 'Enlightenment[13]. The existence of the number line made Cartesian geometry possible, which, in turn, (along with

[8]Previously fractions were only used in practical work. Pure mathematics considered ratios of whole numbers. Three quarters is different from the ratio of 3 to 4. As David Bohm said, "Fragments are not the same as parts."

[9]He did not accept negatives or irrationals, calling them 'useless' or 'absurd'. He did introduce symbolism into his mathematical workings but this was ahead of his time and went no further.

[10]or more detail see Nicola Graves-Gregory, 'Historical Changes in the Concepts of Number, Mathematics and Number Theory', `https://arxiv.org/abs/1705.02386` or Proceedings of Alternative Natural Philosophy Association No.34, July 2014, pp.25–52

[11]Only very few mathematicians (for example, Prof. Welch of Bristol University, and Prof. Ord of Toronto University) are precise in their language, calling it the decimal line since it is based on the now ubiquitous, decimal number system. The decimal line is, of course, a wonderful construction and creates a vast mathematical world but it is not the only mathematical world, and certainly not the real one.

[12]'Humanist' in the sense of elevating the status of human and lessening the status of God, but it gradually became more a materialist philosophy, elevating the concept of dead matter to the previous place of the living God.

[13]The 'Enlightenment' was seen as bringing the light of science, but the shadow it cast on all other forms of knowing, only really began to come to light [sic] in the 20th century.

the development of algebra[14]) made the differential calculus, of Newton and Leibniz possible. With this it was possible to create quantitative equations between changing, measurable, physical observables, which could be used for quantitative prediction. Hail the birth of materialist science! It was no wonder that the mathematics of the next three centuries was dominated by developments in quantitative mathematics generally and the differential calculus more specifically. A positive feedback loop came into being: quantitative description increasingly became the sine qua non for an academic discipline to be considered scientific; and increasingly only 'scientific' research was considered acceptable for funding.

What concerns us particularly here is the acceptance of negative numbers as numbers. It is interesting that historically there was more mathematical resistance to these than to irrationals. Negative numbers make immediate sense in the financial realm from which they come: they represent debt. Money was their ready made home. But as an abstract concept, their ontological status was by no means so obvious. It was only the acceptance of zero as a number that made the concept of the number line possible and thus gave visible meaning to negative numbers. Once this happened (historically this acceptance took many years; now it is taught in primary schools as the norm), then the zero point appeared to be almost arbitrary, just like any other place on the number line extending infinitely in two directions at once; its so easy to just slide over it. But clearly, mathematically, the change from positive to negative is much more than this: its a reversal, a mirror image. Back in the day, bank statements used to emphasise the difference by printing credit amounts in black and debits in red.

There are other, profound implications of the creation of the symbol for zero and its acceptance into the realm of number (which necessitates some special rules so that it does not rock the boat[15]).

[14]The word 'algebra' is also of Arabic origin. The momentous shift around 1600, was from the idea of seeking a determinate, unknown solution to an equation, to the idea of the unknown as a variable. Jacob Klein, 'Greek Mathematical Thought and the Origin of Algebra', transl. Eva Braun. Original German, part l 1934, part ll 1936, English ed'n 1968, Massachussetts Institute of Technology.

[15]For example, that $0/0 = 1$, following the rule $n/n = 1$ for non-zero n. In

One example, coming from the multiplicative level of arithmetic processes, is to think of a potential zero, a complement of potential infinity. This is the polarity of expanding our minds to the far reaches of our comprehension of the vast universe and deepening our minds to the far reaches of our comprehension of our inner reality. But such implications do not concern us here. They have not filtered through into general consciousness to alter the sliding scale view of the number line at the +/- level. This view is now embedded in cultural thinking, resulting in the subconscious idea that all antinomies can be seen in this way.

Prof. Zeki's research shows that, in the case of the beautiful and the ugly, it is decidedly not so. They are not on a sliding scale of comparison. They are discrete concepts, of different natures: they activate different parts of our brains. So Prof. Zeki's work shows us a way of developing our humanity, not by creating comparative scales of beauty and ugliness in any sphere[16], but rather by developing our appreciation of beauty per se. This leads us back to Plato and to Pythagorean mathematics where the ontological difference between whole and divisible is fundamental.

Clearly we are now entering that aspect of *arithmetike* which has been excised from mainstream mathematics, the linkage with human, social concepts – taboo! For the Pythagoreans, there were direct correspondences between specific integers and specific concepts. We know little about these, so the general denigration of this, is inappropriate. There may have been other forms of linkage, and perhaps, in any case, we might bring some into being. For example, beauty, is a good candidate for a meta-mathematical concept, one that belongs to the philosophy of mathematics.

This is certainly the case when we interpret philosophy in its original, Pythagorean sense, 'wisdom of love'[17]. Our normal mean-

common sense terms dividing zero by zero is meaningless. Dividing a finite number by zero, in the common sense way, 'how many times does 0 go into n', does come up with infinity. We are seeing here the extension of the concept of number being an essential part of the seismic shift of mathematics into a symbolic language, floating free of verbal language, a language with its own grammar, logic and generating principles.

[16] "The better is the enemy of the best" is an age old adage.

[17] *'Philia'*, love; *'sophia'*, wisdom. The structure is the same as other Greek

ing of philosophy relegates it to mere, epiphenomenal, descriptive activity. The original, Pythagorean, wisdom of love, was the active principle that gave birth to mathematics. This term, *'ta mathemata'*, coined by Pythagoras, means 'those things that have been learned'. The community was primarily a spiritual one, in fact an esoteric[18] one; this learning was holistic, spiritual, abstract and material.

We can posit (or imagine) a Pythagorean, meta-mathematical, conceptual realm, one with guiding principles for the development of mathematics. For Plato, the ideals of beauty, truth and the good were precisely the guiding principles to ascend to the Divine. Cicero, 1st century BC philosopher and Roman statesman, adopted them as part of his programme of education for Roman citizens to be civilised, namely the Seven Liberal Arts. These comprised the quadrivium of Pythagorean mathematics: number, geometry, music and astronomy, and the trivium of verbal arts: grammar, logic and rhetoric.

We know that, for nearly all mathematicians, the beauty of mathematics is paramount. Dirac said that "It is more important to have beauty in one's equations than to have them fit experiment." His beautiful, eponymous equation was an example of this: it seemed outlandish at first; it was only later that it was recognised as being key in quantum physics. Mathematics is now also almost universally considered to be a repository of truth. Bertrand Russell referred to them together: "Mathematics rightly viewed, possesses not only truth but supreme beauty..."[19]. What about the good? Given that in normal parlance, we associate the beautiful with good, and the ugly with bad, it seems very likely that contemplation of the good would stimulate the frontal cortex, whereas focussing on the bad would activate the more primitive, amygdala

portmanteau words, 'geometry', *'ge'* is 'earth' and *'metrein'* is 'to measure' etc. Note: "Pythagoras is said to have been the first to call himself 'philosopher' ". Iamblichus, full ref. note 7, p.70

[18]Our present meaning of 'esoteric' as 'mystical', derives from the fact that this was the nature of the inner group of Pythagoreans: *'esoterikos'* in Greek means simply 'inner'.

[19]Bertrand Russell, (1919). p.60, 'The Study of Mathematics' in 'Mysticism and Logic: And Other Essays'

and motor cortex. And, in fact, this is also, probably true [sic] for truth and falsehood.

There are other antinomies that might well give the same neural effects. Masaru Emoto's experiments with water, are interesting in this regard. For example, he placed the written word 'Love' under a glass of water for some time, and the written word 'Hate' under a different glass of water for the same length of time. He then slowly froze the water on Petrie dishes so that snowflake like crystals were formed. The 'Hate' imprinted ones were misshapen; the 'Love' imprinted ones manifested beautiful symmetries[20]. Theodor Schwenk developed a similar method for testing the quality of drinking water, through drop pictures[21]. As we know our brains, our whole bodies, are mainly water!

The quantitative mathematics of the calculus has been used as the servant of materialist science, to great effect, to create machines to control and destroy, both things and people[22]. Could we create a 21st century Pythagorean mathematics that helps us develop our humanity through developing our appreciation of beauty and truth and the good? What might it mean in practice, to develop our appreciation of, awareness of, beauty? And what might a mathematics of beauty be?

When we see or hear or otherwise behold something that causes us to think or say, "This is beautiful!", we are experiencing beauty. Our relationship with this other gives rise to this experience in us. Relationships are the essence of mathematics[23]. In our existing, mathematical terminology this relationship could be represented as some kind of mapping or projection from ourselves to some other. If the other is human, then we might ask what might the nature

[20]Masaru Emoto, 'The Hidden Messages in Water', publ.2004 'Beyond Words', Simon and Schuster Inc.

[21]Bertram von Zabern, 'Organic Physics' publ. Mercury Press 1995, p. 228 3rd Ed. 2020

[22]Shoshanna Zuboff in 'The Age of Surveillance Capitalism', publ. 2019, Profile Books Ltd., London (also on Youtube) gives an example of Facebook, offering companies a system whereby Facebook informs them when young people are feeling low so they can send an advert at that time, "You always feel good in jeans like this" or similar.

[23]Perhaps it is not so surprising that the Pythagorean path of *philia*, friendship, should give birth to European mathematics.

of the return mapping be. If the other is non-human then, from our normal, consensus reality perspective, the projection from the human would be considered a one-way mapping.

This is not the only possibility. The poet, Rainer Maria Rilke, in the eighth of his 'Duino Elegies' is wholly concerned with different natures of relationship between ourselves and the world we see and are part of:

> "...Never, not for a single day
> do we have pure space before us in which the flowers
> are endlessly unfolding. It's forever world
> and never Nowhere-without-Not:
> the pure and unwatched-over which we breathe
> and endlessly *know* and do not desire..."[24]

The theologian Martin Buber's endeavour is to explicate the difference between 3rd person and 2nd person living: the difference between the relationship, I and Thou, and the relationship, I and It. He writes, for example, of meeting a tree as Thou[25]. Stephen Harrod Buhner gives numerous examples of shamans in relationship with plant spirits as their teachers.[26].

According to the current, mathematical paradigm, such views are extra-mathematical. But we are beginning to consider a 21st century Pythagorean, holistic mathematics which is broader and deeper than the quantitative, symbolic discipline that we mostly grew up with. It is important to realise that the fully symbolic language of mathematics, which is now the norm, only emerged in the watershed of the 16th and 17th centuries AD. Before that, mathematical ideas were embedded in verbal language. We are only just beginning to envisage a phoenix of Pythagorean mathematics; it would be unreasonable to expect to be able to create

[24]Rainer Maria Rilke, 8th Duino Elegy, p.61 in 'The Duino Elegies', written between 1912 and 1922, transl. C.F.Macintyre, publ. Dover Publ'ns Inc., NY, 2007

[25]Martin Buber, 'I and Thou', authorised transl'n, Ronald Gregor Smith 1937, from 'Ich und Du', Berlin 1923, 2004 ed'n publ. Continuum, London, UK, pp.14–15.

[26]Stephen Harrod Buhner, 'The Lost Language of Plants', publ. Charles Green Publ. Co., USA, 2002. For example p.8.

immediately a fully fledged symbolism[27]. Nevertheless, steps in this direction have already been taken by a very remarkable mathematician and computer scientist, Françoise Chatelin[28], unfortunately recently deceased.

Already in the 20th century the beginning of the end of the Baconian dream, (characterised by his dictum, loosely translated as "Knowledge is power"[29]) was in sight. Francis Bacon believed that the new materialist science (not his naming), which was emerging at the turn of the 16th to 17th centuries, would make life easier for all people. This has been the belief of nearly all generations of scientists and politicians in the four centuries since then, when they've heralded in a new technological era. And each time, as we have seen, this dream has not manifested. Instead, inequality of wealth has continued to increase. In 2020 the majority of the population on this planet, struggle to make a living. One per cent of people control two thirds of the wealth. We could now be at a turning point, not because the wealthy are giving up their wealth, but because more people of the ninety-nine per cent have noticed the extent to which our wonderworking technology has been polluting and ravaging the planet in all spheres, earth, water, air and electromagnetic radiation[30], and are calling for change.

It is the quantitative mathematics of the differential calculus that has made possible both the tremendous technological progress of the last three centuries (at a high cost), and the extreme sophistication of capitalist economics which has given free rein to bankers' greed, precipitating crashes which have devastated the poor and middle classes and left the rich untouched. Francis Bacon's mistake, repeated by similarly motivated idealists after him, was to think that the problem was purely material. It is for this reason

[27]In fact, part of 21st century Pythagorean mathematics will include reviving the vital distinction between sign and symbol. See NGG 'Towards a 21st century Pythagorean mathematics' in this volume of ANPA Proceedings.

[28]Françoise Chatelin, 'Qualitative Computing: A Computational Journey into Nonlinearity', publ. World Scientific, 2005, also in this ANPA Proceedings

[29]Francis Bacon, "ipsa Scientia potestas est." in his 'Meditationes Sacrae' 1597

[30]For example, deforestation, plastic in the oceans, holes in the ozone layer and electrotechnical communication pollution.

that it is clearly time to return to the holism of Pythagorean mathematics and to ask ourselves what other kinds of mathematics are possible. Interestingly, in the 20th century more qualitative mathematics began to emerge[31].

Newton and Leibniz could not foresee how their synchronous creations of the differential calculus would be used and abused. Pythagoras was clear that his mathematics could be used for good or evil and tried to ensure that it would only be used responsibly for good purposes, but to no avail. Note that all three were deeply spiritual people and Pythagoras and Newton, in fact, considered their spiritual work primary. Newton believed he would mainly be remembered for his work on the Book of Revelations. For both Pythagoras and Newton, the goal of understanding material reality better was to appreciate this as Divine creation. And one could only understand Divine creation better through being more aligned with, attuned to, the Divine process: one had to be willing to change, to be changed, in the process of attempting to understand. For Newton this was his alchemical practice[32]. For the Pythagoreans, it was philosophy, the 'wisdom of love'.

Note that Newton's practice was a solitary, individual one, whereas Pythagorean love was that between friends. There was a different sense of the individual in the time of the Renaissance and Reformation from that in ancient Greece[33]. The *'philia'* of *'philosophia'* is 'loving friendship'. The other Greek words for love are *'eros'*, 'erotic or romantic love' and *'agape'*, 'love of the Divine'. It's interesting that the spirituality of Pythagoreanism, the ascent to the Divine Oneness, the Monas, is through human loving friendship. The similarities with later Christian teaching are striking[34]. It's also interesting that in Plato's 'Symposium', the type of love

[31] For example, topology and the 4 Cs, cybernetics, catastrophe theory, chaos theory, complexity theory.

[32] Michael White, 'Isaac Newton, the Last Sorcerer', publ. 1997, Fourth Estate Ltd., London UK

[33] Ernst Cassirer, 'The Individual and the Cosmos in the Renaissance', transl. Mario Domandi, publ. Univ. Pennsylvania Pr., Philadelphia, 1963. 1st publ. in German, 1927, Volume X of 'Studien der Bibliothek Warburg'.

[34] Pythagoras' teaching fed into Platonism which strongly influenced the development of Christianity.

being discussed, is *'eros'*. Pythagorean love, *'philia'* is broader.

The possibility of meeting the Divine in or through another human being is, in fact, now thought of mainly as heightened eros. Rilke describes this in his 'Duino Elegies' [35] But it is also possible in friendship as is acknowledged in the Sanskrit greeting, *'namaste'*, 'I honour the Divine in you'. And Martin Buber is clear that it is possible for us to revere the Divine in all life and it is only our materialist perspective that obscures this, reducing all life to dead *'It'* [36].

Whether the way is through alchemy or friendship, the goal of these epoch-changing mathematicians is the same, meeting the Divine. Rilke sees that this is where our experience of, or relationship with, beauty, leads us, and sees how we fear it: "For beauty is nothing but the beginning of terror which we can only just bear and we wonder at it so, because it serenely disdains to destroy us. Every angel is terrifying."[37]. T.S.Eliot, in a more contemporary vein, wrote, "Humankind can not bear very much reality"[38]. Beauty brings us close to true reality, within and behind our normal, safe, consensus reality.

Of course, our normal, consensus reality is necessary for our material, social living. But the source of this miraculous universe, this extraordinary life is beyond this. Believers in materialism, i.e. those who believe that materialist science is capable of explaining all the wonders of the universe, ask that the Big Bang (the occurrence of which is beyond materialist scientific explanation) be considered a fact, not a miracle. They also claim that everything, including human consciousness will be explained in wholly mate-

[35]Rilke, in the 8th of his 'Duino Elegies' (Note 23 edn) p.63, "Lovers, were it not for the other who blocks the view, are close to it and marvel ... as if by carelessness it is open to them behind each other ... but neither gets past and again it's world."

[36]Martin Buber, 'I and Thou', authorised transl'n, Ronald Gregor Smith 1937, from 'Ich und Du', Berlin 1923, 2004 ed'n publ. Continuum, London, UK. This is the whole tenor of his writing.

[37]Rilke, '1st Duino Elegy', (full ref., note 23) p.2, *"Denn das Schöne ist nichts als des Schrecklichen Anfang, den wir noch grade ertragen, und wir bewundern es so, weil es gelassen verschmäht, uns zu zerstören. Ein jeder Engel ist schrecklich."*

[38]T.S.Eliot, 'The Four Quartets, Burnt Norton'

rial terms at some time. After 100 years of neuroscience (similar to cutting open torsos, in earlier times, to find the soul) they are no nearer. This is not surprising since consciousness is of a different nature from matter.

Martin Buber is clear: the consciousness which remains locked in normal, consensus reality, is living in the realm of *It*, as distinct from the realm of *Thou*[39].

In the realm of *It*, everything is a thing; all experience is reified, including beauty. So, beauty is not experienced as a revelation, the brink of an awesome (in the correct use of the term) encounter; instead it is reduced to a mere, human assessment. The word, 'beauty' has been demeaned, trivialised: people speak of beautiful shoes, food, make up etc. Nevertheless, real beauty is still a human need. We go to art galleries, museums, concerts and to nature and the wild, to experience beauty, but mostly in these situations, we are consumers; we just get a taste [sic], we eat beauty, we do not enter her; we do not meet her.

Meetings in mathematical terminology are incidences, where two lines cross. Meetings with beauty, longed for and feared, could be seen as meetings of parallel lines at infinity. In Greek times, as we know from Euclid, parallel lines never met. The Pre-Socratic, mystic philosophers like Pythagoras, went into another world space, often an underworld space to receive Divine wisdom[40].

In the Renaissance when perspective came into visual art, the point at infinity where all parallel lines met, was visible on the canvas. Did this change the nature of alchemy? Was it more possible from then, for alchemists like Newton to envisage meeting the Divine from a conscious space? At one time scholars assumed that Newton engaged in alchemy after he had completed the Principia. This accorded best with their logical positivist view of history: they could dismiss the alchemy as a product of Newtons dotage when his

[39]Martin Buber, 'I and Thou' authorised transl'n, Ronald Gregor Smith 1937, from 'Ich und Du', Berlin 1923, 2004 ed'n publ. Continuum, London, UK. Again, this is the whole tenor of his writing.

[40]Peter Kingsley, pp.209–214, 'In the Dark Places of Wisdom', publ. Elements Books Ltd, UK 1999.

mental powers were less acute. But Betty Dobbs' research[41] showed that this was not the case. Newton's alchemy and his mathematical and scientific work developed hand in hand. Similarly, Pythagoras' philosophy and his mathematics.

We have begun to look at a possible 21st century Pythagorean mathematics primarily as a meta-mathematics of guiding principles but it may, as with Newton, lead to what we now consider more mathematical domains as well. As mentioned, Françoise Chatelin's work has begun this. Certainly, for the 6th century BC Pythagoreans, the qualitative study of number, *arithmetike*, was complementary to *logistike*, the study of calculating with number as quantity. The concepts of odd, even, prime, composite, triangular, square, perfect etc., which make up the characteristics of the different integers, come from *logistike*. Srinivasa Ramanujan was renowned for his relationship with hundreds of integers, through knowing these and more complex characteristics[42].

The *logistike* of modern mathematics is now so extensive that it is very difficult for number theoreticians to span enough disciplines to get such an overview. It was only through his totally dedicated focus that Andrew Wiles, was able to prove Fermat's last theorem, a number theoretical theorem which thousands of mathematicians (and amateurs) had studied fruitlessly since Fermat's time. It was Wiles' lifetime's work from when he was a schoolboy, following a thread he did not share with other mathematicians[43], studying seemingly disparate disciplines, which eventually revealed themselves to be connected in his extremely complex proof, thereby enriching mathematics immensely.

We have begun to consider how our experience of beauty could be seen as some kind of mapping between ourselves and the other,

[41] B.J.T.Dobbs, 'The Foundations of Newton's Alchemy or The Hunting of the Green Lyon', Cambridge University Press 1975.

[42] G.H.Hardy remembered going to see Ramanujan when he was ill at Putney, saying, "I had ridden in taxi cab number 1729 and remarked that the number seemed to me rather a dull one, and that I hoped it was not an unfavourable omen." "No," he replied, "it is a very interesting number; it is the smallest number expressible as the sum of two cubes in two different ways."

[43] John Horton Conway (personal conversation) told of how the other mathematicians kept wondering what Andrew Wiles was doing. Mathematicians generally discuss their work a lot with each other.

or as a meeting between two lines. As Martin Buber says, 'All real living is meeting'[44]. Can the incidence of two lines evoke or invoke that sense of real living? Well, yes, when we re-member that the meeting is a point and that the point is an ideal, spiritual entity, a living idea which is never realised in the material world because it has no size. This is the potential zero which draws us in and in. It is the point moment of Zen, the open secret in Zeno's paradoxes.

It also arose in materialist science at the beginning of the 20th century in one of their beautiful stories of the invisible[45], namely, quantum physics. When we go 'down the rabbit hole'[46] into the small, small, small, we enter a wholly different world where the space, time, material laws of consensus reality no longer hold sway. This is the point, the meeting point, the point of the meeting, the glimpse of the Divine, the miracle of the gift of this world, the gift of life. This is the experience of the beauty, and of the truth, of life. This is what Keats recognises, "Beauty is truth, truth is beauty. This is all we know and all we need to know". Schiller goes further: "Only through Beauty's morning-gate, dost thou penetrate the land of knowledge". The question then arises: what kind of knowledge[47]? T.S.Eliot goes further still and questions our desire for knowledge per se, "What knowledge have we lost through gaining information? What wisdom have we lost through gaining knowledge?".

As we know, our quantitative mathematics has helped us accrue vast stores of information, and much knowledge of the external world, as we perceive it through our object-focussed perspective,

[44]Martin Buber, 'I and Thou', authorised transl'n, Ronald Gregor Smith 1937, from 'Ich und Du', Berlin 1923, 2004 ed'n publ. Continuum, London, UK, pp.17.

[45]One quasi-definition of mathematics could be a conglomeration of languages to describe invisible and intangible worlds.

[46]As in 'Alice in Wonderland' but a different kind of wonderland from the one that Lewis Carroll told us of.

[47]'Knowledge' is generally 'knowledge about things'. This is the sort of knowledge that can be accumulated, like money and tested in exams. But there are other kinds of knowing, knowhow, knowing-when-to. In other languages there is a separate verb for the vital knowing of living relationship, 'kennen' in German, 'cognoscere' in Italian etc. See NGG, 'God be in my Heart and in my Thinking: Towards Self-awareness in Mathematics and Science', pp.202–223, 'Contexts', ANPA Proceedings 31, 2011.

as *It*. This is knowledge based on lesser truth, legal truth, truth that has been equated with fact, which is a nonsense, since we know that facts are always contextual whereas the truth that Keats knew, is transcendent.

When positivism replaced Platonism as the dominant philosophy of mathematics, truth came to mean merely a formal recognition of a statement which had been logically proven. But in fact, most mathematicians are closet Platonists[48] and, as we've seen, Dirac chose beauty to lead him to truth in his scientific work, rather than attempting to bend to the facts as they appeared at that time.

T.S.Eliot re-minds us of the possibility of the call of wisdom. This is the wisdom of acknowledging Thou, in the point where parallel lines meet, where beauty and truth coincide once they are returned to their original purity and potency. This is the truth spoken of in the Vedas where *'sat'* means both 'truth' and 'being'. This is the 'aha' moment when we understand a mathematical theorem or proof. It was on this experience that Husserl based his phenomenology of mathematics. This, like the qualitative mathematics which began to appear in the 20th century, is a harbinger of the renaissance of Pythagorean mathematics, now both needed and possible. The heart of this is is the truth of Pythagoras and Plato, the truth of integrity.

So, what about the third component of Plato's trinity, the good? 'The good', as such, is no longer in common parlance. There is 'being good', i.e. conforming to a set of rules. 'Looking good' just means 'attractive' 'Doing good' does have a higher motivation than these but it still does not convey the aspiration in the original Platonic usage, to the good which is the deep wholeness of the universe. It has a more limited sense of altruism, doing something for others. In the business world the underlying motivation is often in order to 'look good'. It has not been seen as an integral part of the focus of mathematics for centuries, because mathematics has been considered neutral, value-free. A few mathematicians

[48]There is a similarity here to the situation of Christians in Communist Russia.

have been concerned about the way mathematics is used and the extent to which mathematicians need to take responsibility for the results of their mathematics. Alexander Grothendieck famously left the Institut des Hautes Etudes Scientifiques (IHES) in 1970 on learning of its military funding. More recently, Tom Leinster, of Edinburgh university, has initiated conversations about this, for example, questioning implications of working with GCHQ[49]. Has Shoshanna Zuboff's work raised questions in more mathematicians' minds?[50].

Is 'the good' linked with beauty? In faery tales, the heroine or hero is nearly always not only good but also beautiful. The intention of these stories as childhood nourishment for the soul, is to impart the sense of the beauty of the upright person, the good heart and the moral deed[51]. But in many people's minds, good is associated with physical, good looks. Good-looking people can get away with more, because most people believe that they are good, whereas people who do not fit societal ideas of good-looking, tend to be less trusted. There is sociological evidence for this response[52].

Are we in a position to envision a mathematics of beauty as part of Plato's trinity? One mathematical, ideational process is to create geometric forms representing relationships between concepts. For example, the Chinese yin-yang symbol can represent complementary pairs mathematically. Fish and chips together make the circle, a new whole, the classic British take-away! Similarly, the Platonic trinity of beauty, truth and the good can be seen mathematically in the gestalt of a triangle.

I think there is another mathematical form that represents even

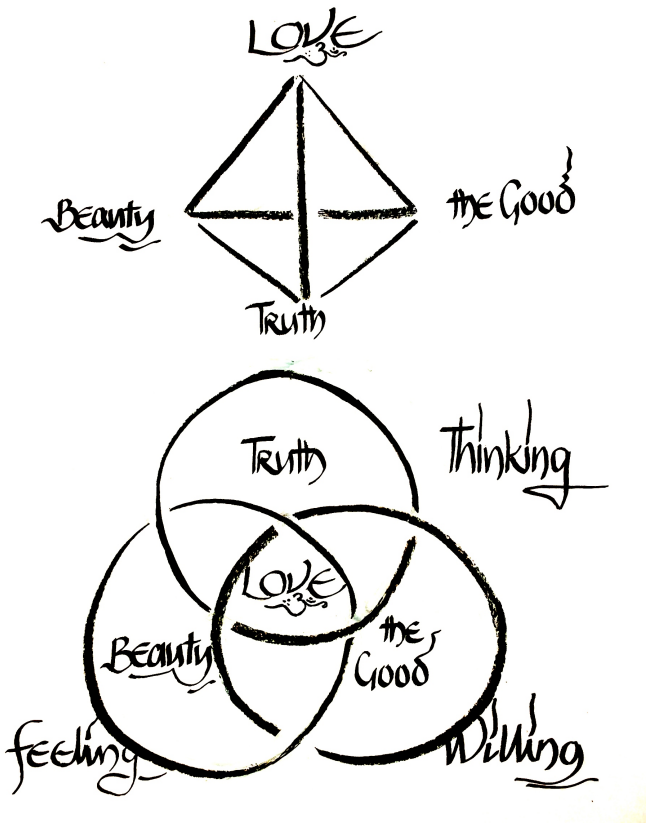
[49]Tom Leinster, Should mathematicians cooperate with GCHQ?' `https://classes.golem.ph.utexas.edu/category/2014/06/should_mathematicians_cooperat_2.html`

[50]At present, quantitative mathematics is the most powerful, amoral entity operating in the human world, manifesting in the global, capitalist economy. Surely such an amoral entity needs some kind of moral restraints? NGG, book-in-progress, 'Mathematics and Morality, Refining Awareness, Deepening Love'.

[51]Bruno Bettelheim, 'The Uses of Enchantment, The Meaning and Importance of Fairy Tales', 1st publ. 1976, Thames and Hudson UK.

[52]`https://www.psychologytoday.com/us/blog/so-sue-me/201408/do-attractive-people-fare-better-in-the-courtroom`

more closely the unity of this trinity. The three Platonic ideals correspond to the three fundamental, human modalities that Rudolf Steiner identified: thinking, feeling and willing. In thinking we aspire to truth; in our will, we aspire to the good; in feeling we develop our appreciation of beauty. Our present day study of beauty, aesthetics, takes its name from the Greek, *'aisthesis'*, meaning 'perception by the senses, especially by feeling'[53]. In the Borromean Rings, (see the diagram), the three circles represent in turn, thinking and truth, feeling and beauty, willing and the good. The three together are strong. If any one circle is unlinked, the whole falls apart. Unfortunately, this is what happened.



[53]Greek-English Lexicon, Liddell and Scott, Oxford University Press, 1963, p.21.

There is hope, if we follow the Pythagorean path, that we might realign our thinking, feeling and willing, to their highest ideals, by placing Love in the centre, as their focus. The word 'focus' comes from Latin, meaning, 'hearth fire'. It was Kepler who first used the word 'focus' for the point of convergence of a lens, probably because it is the lens' burning point. This brings to mind yet another mathematical form, namely the tetrahedron. This represented fire for Plato when he related the elements, fire, earth, water, air, ether, with the five perfect solids (see the diagram). Here Love, the burning point is the apex of the pyramid, rising above the equilateral triangle of the Platonic ideals, beauty, truth, the good.

This gives rise to another possibility, returning to the image of the Borromean rings. The pairwise intersections could be given meanings. Beauty and the good could manifest as art; truth and beauty as mathematics; truth and the good as politics: ideals of a civilised society to which we could aspire. Note the imaginary nature of this image. The intersections correspond to a Venn diagram more than to Borromean rings. We have much work (and play) ahead when we research different kinds of imaginary and/or imaginal and/or spiritual worlds.

There is hope also, in the good sense of the young who have not even heard of Aristotle, Plato or Pythagoras. My godson, Ben, aged 7, knows true beauty. When I went round to see him, shortly after Christmas 2018, Ben was very excited about a Lego dragon he'd been given, a magnificent construction, all transparent plastic, complex and dramatic, with shiny, blue and turquoise blades for wings.

"He's beautiful.", said Ben, "At least, he would be beautiful – if he was good – but he's not!"

Ben is a Platonist! Thanks be.